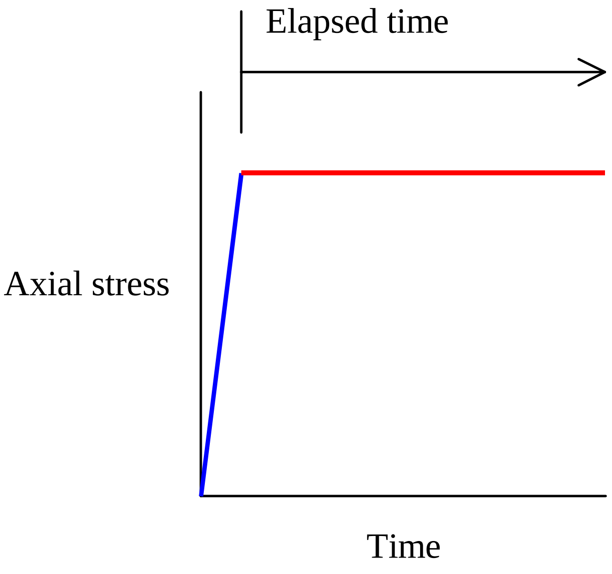
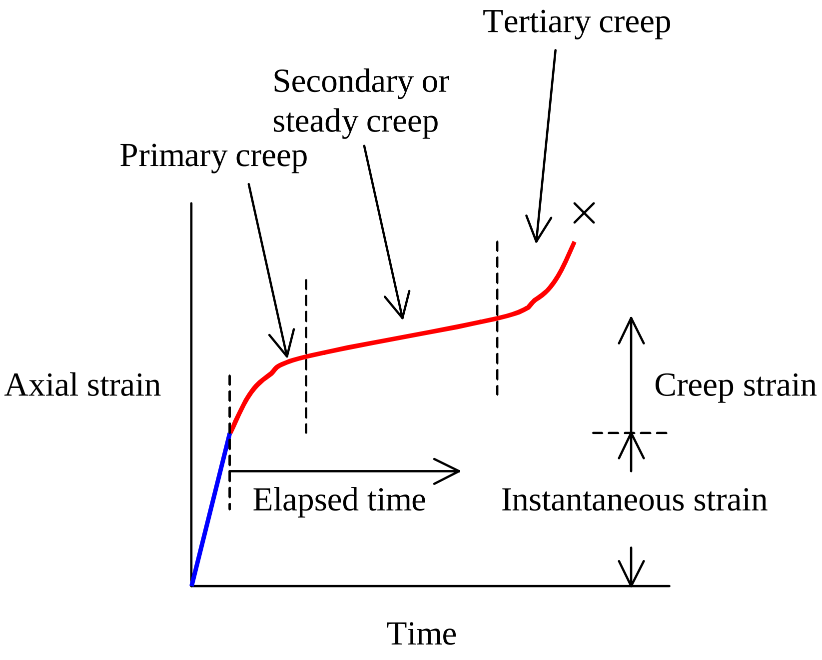
6. Creep and fatigue failure

6.1 Creep as a phenomena

Creep

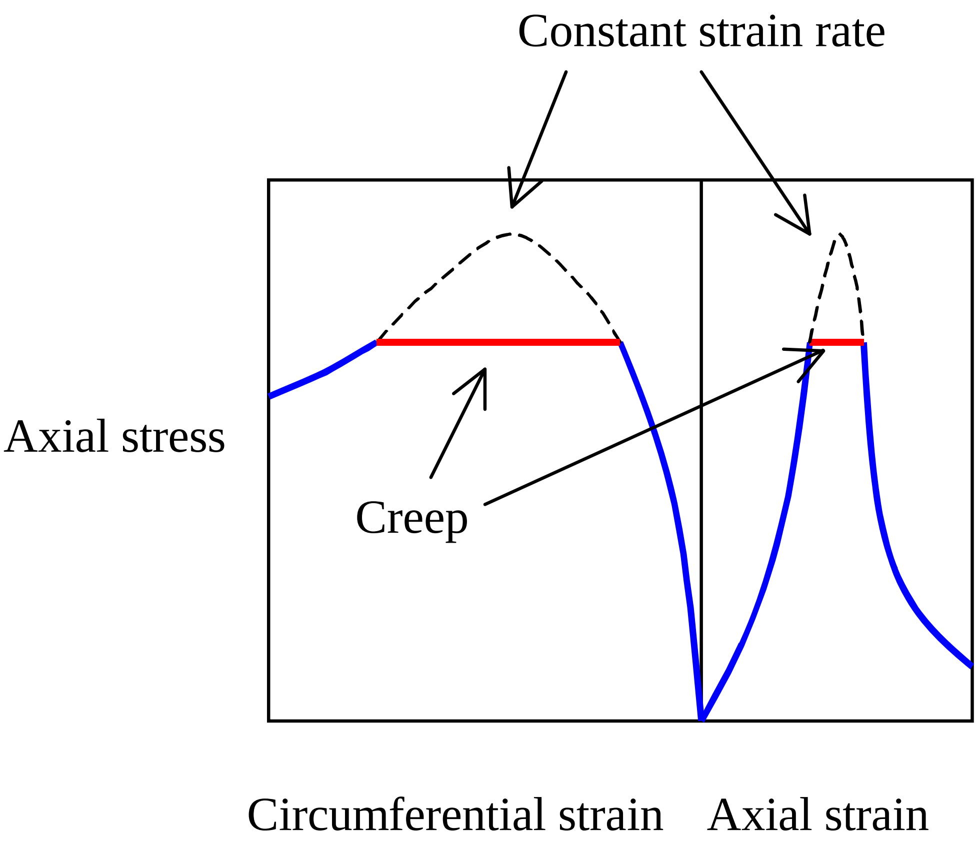
- Strain increases under a constant stress



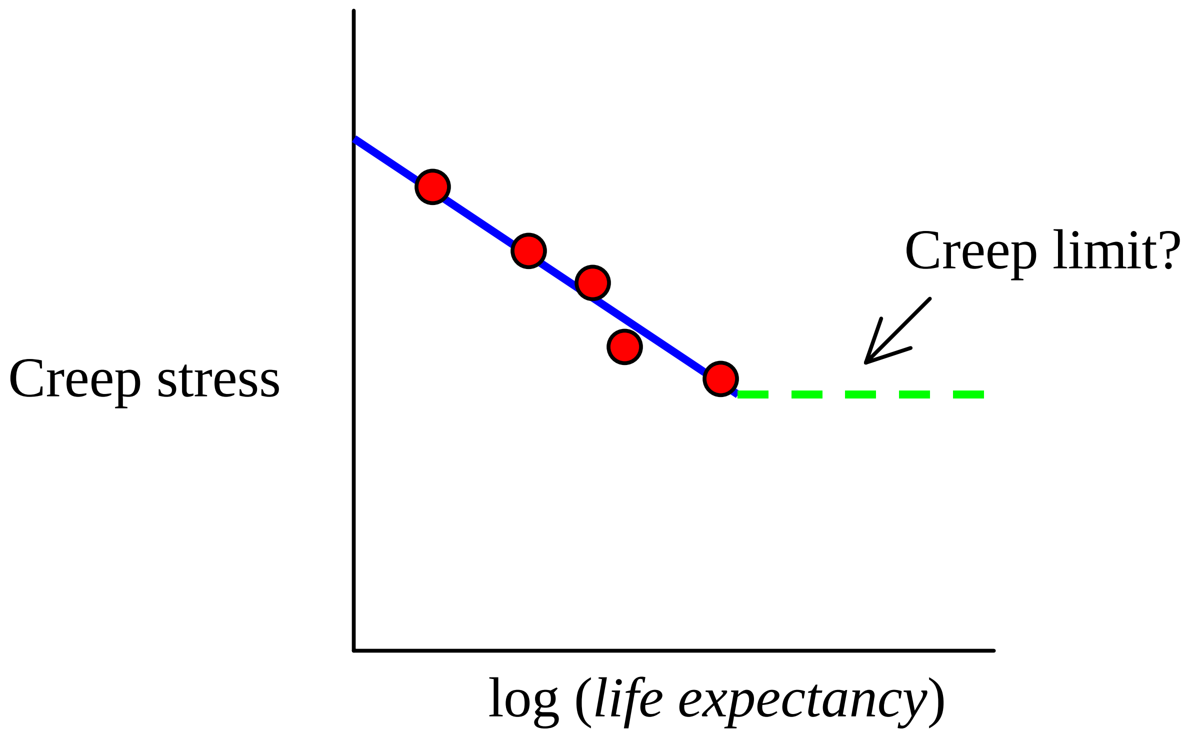


Why creep?

- Failure of rock mass around tunnel
- Failure of rock slope
  - with trigger
    - pore pressure, thermal stress, freeze thaw cycle, weathering, etc.
  - without trigger
    - creep


Constant strain rate
Axial stress
Creep
Circumferential strain
Axial strain

Creep stress and time to failure



- Q: Time to failure of a rock is 3.6×10^3 s and 3.6×10^4 s under 100 MPa and 80 MPa of creep stress, respectively. Estimate time to failure under 85 MPa of creep stress.
- A： 20230 s

Strain rate in secondary creep

$$\dot{\varepsilon} = A\sigma^n$$

$\dot{\varepsilon}$: strain rate

A : constant

σ : creep stress

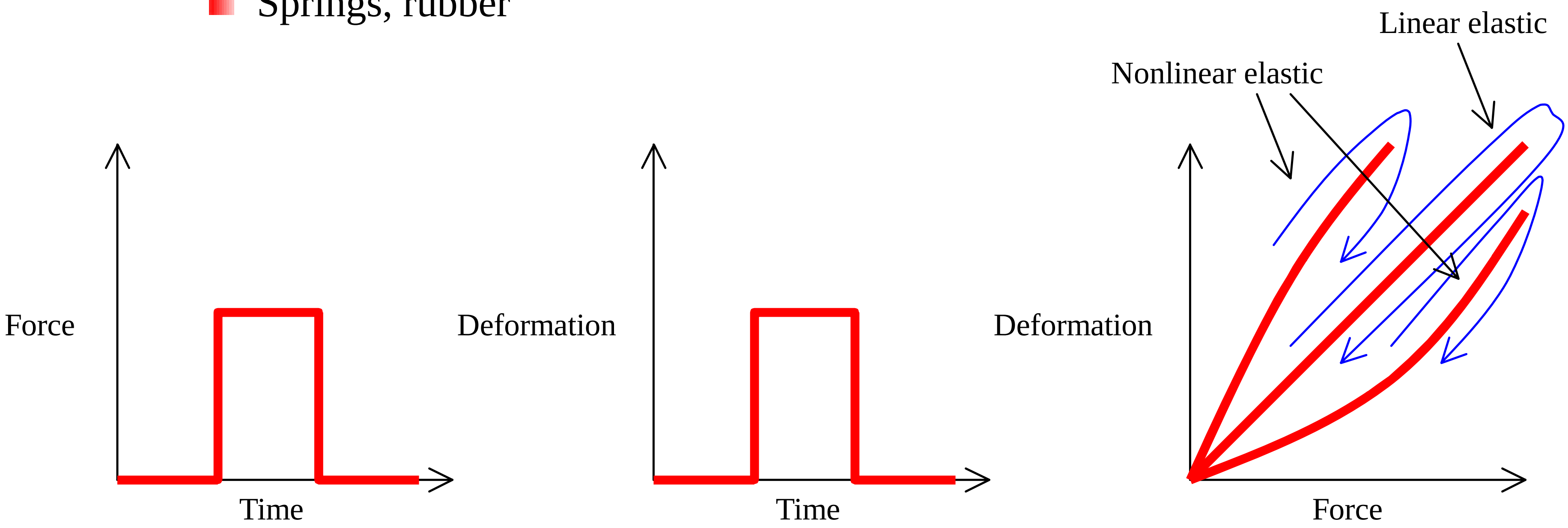
n : constant (20～50)

6.2 Mechanical properties of bodies

6.2.1 Elasticity, viscosity and plasticity

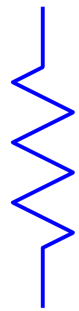
Elasticity

- Elastic body deforms when subjected to force
- Deformation vanishes when force is removed
- Example of elastic bodies
    - Springs, rubber

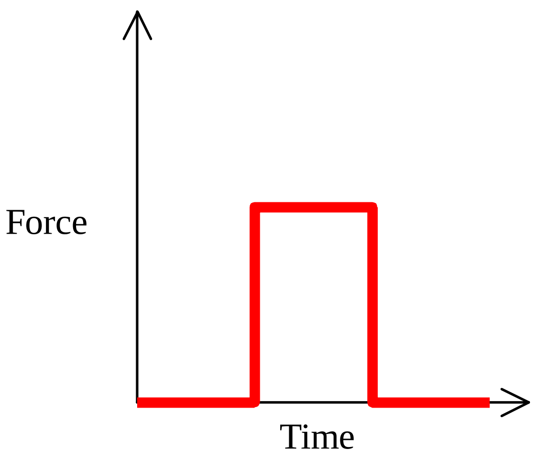


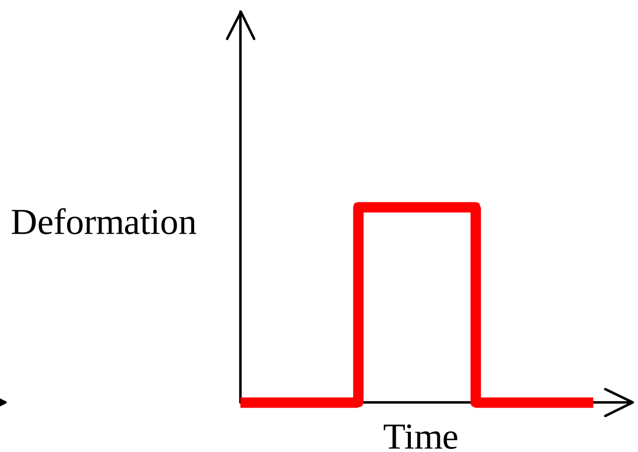
Linear elastic body

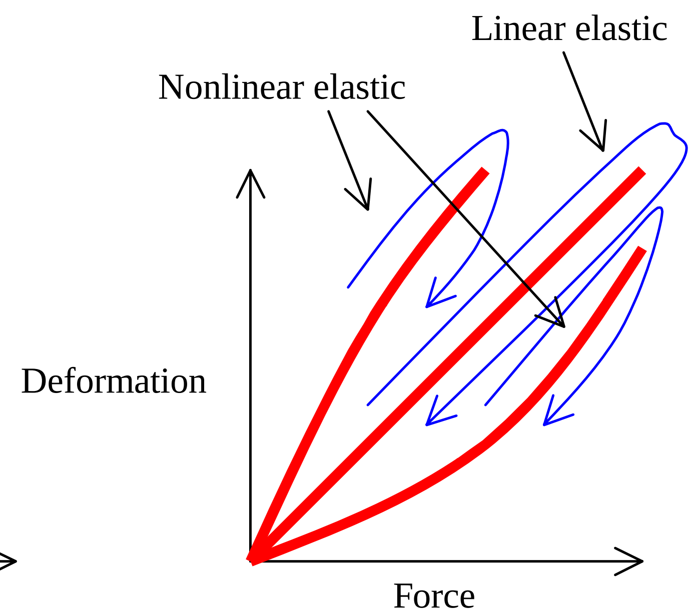
- Spring is used as an element body



$$\varepsilon = \frac{\sigma}{E}$$

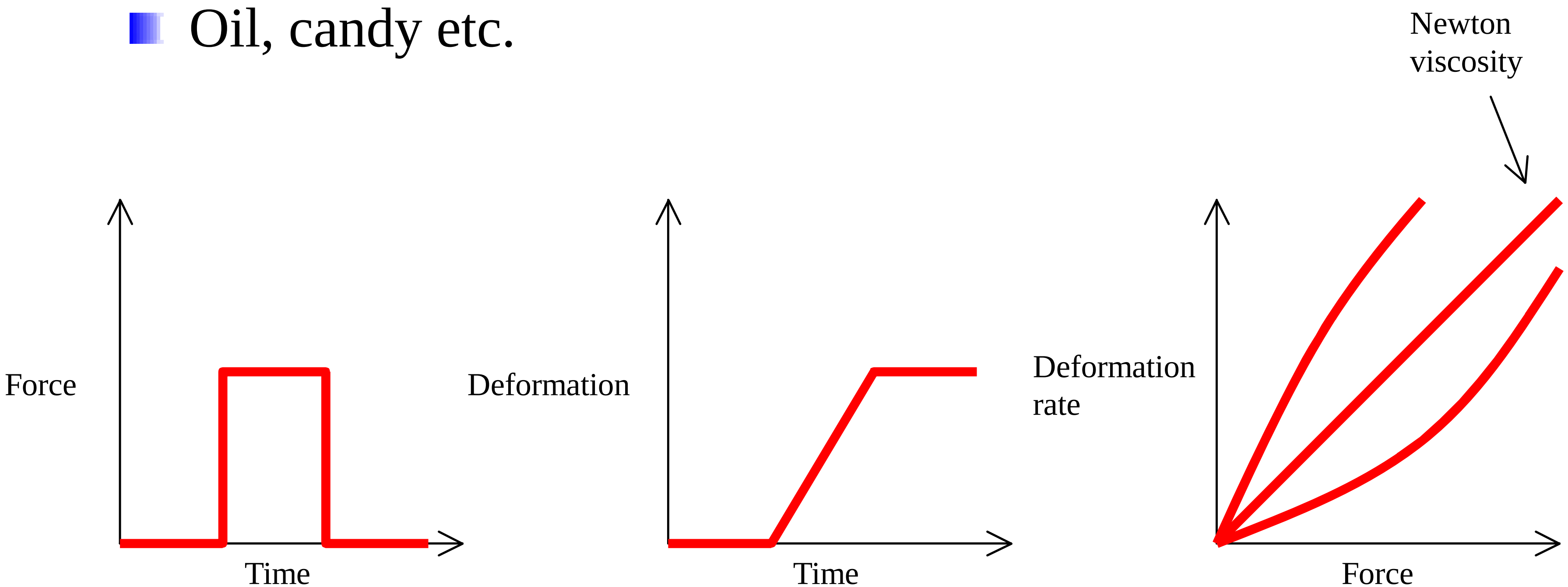




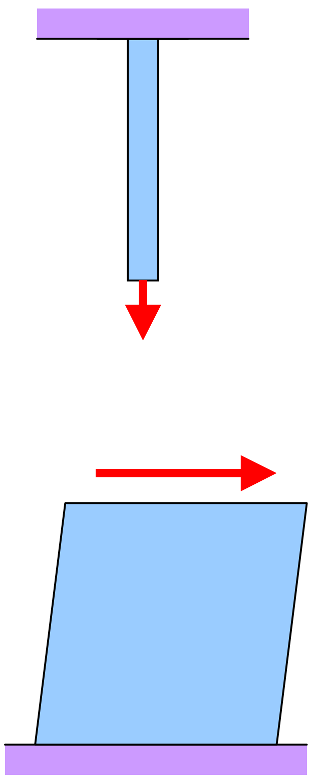


Viscosity

- Deformation continues under constant force
- Deformation does not vanish when force is removed
- Oil, candy etc.



Newtonian body



$$\sigma = \eta \dot{\varepsilon}$$

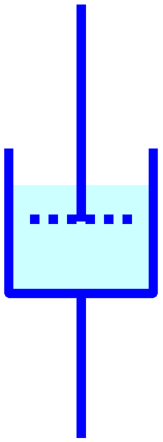
σ	Normal stress
η	Normal viscosity
$\dot{\varepsilon}$	Strain rate

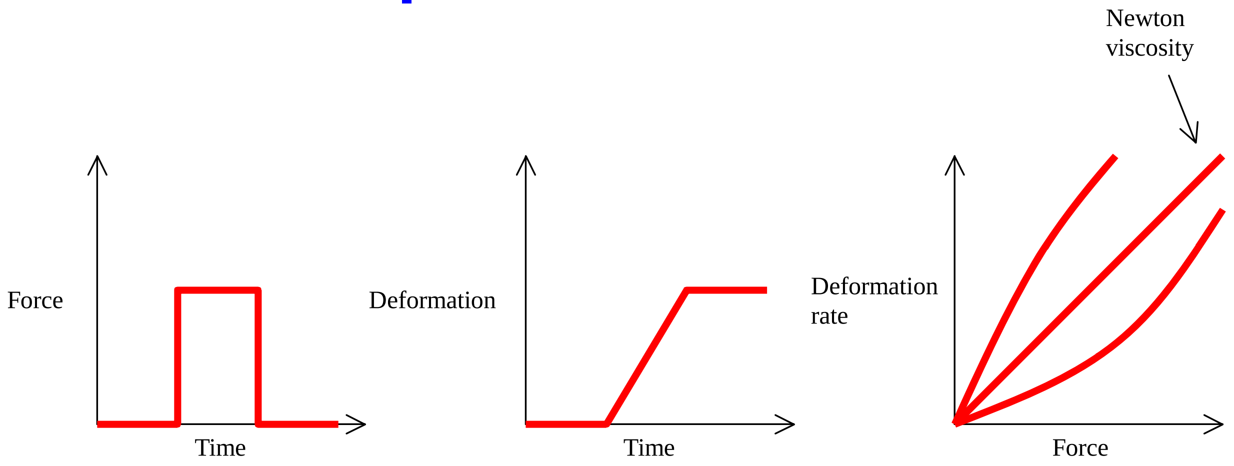
$$\tau = \mu \dot{\gamma}$$

τ	Shear stress
μ	Viscosity
$\dot{\gamma}$	Rate of shear

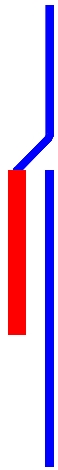
An element body for Newtonian body

- Dashpot is used



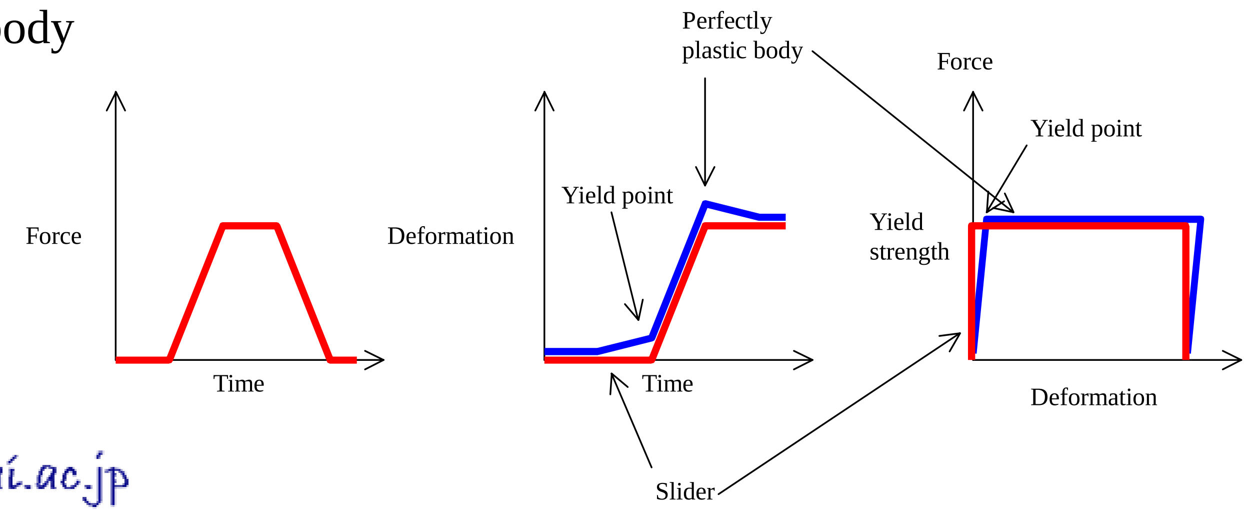


Plasticity

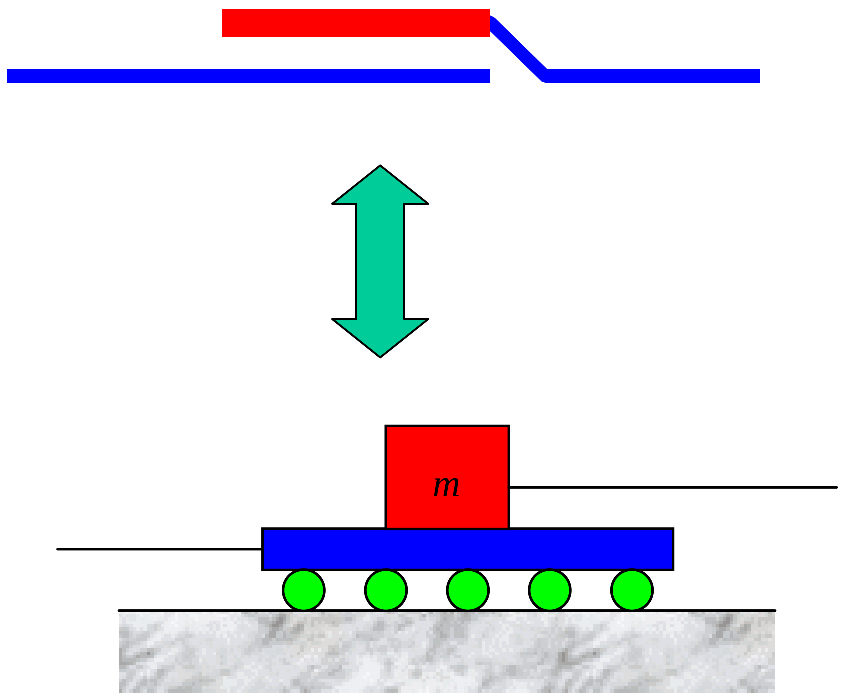


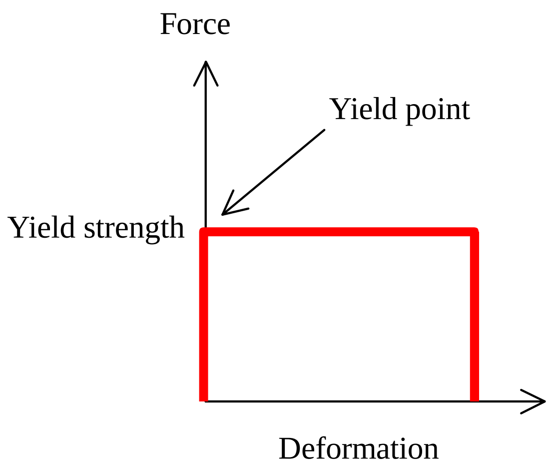
- Yield and deformation occurs at a stress
- Deformation does not vanish when stres is removed
  - Clay etc.

- A body which does not deform until yield point
  - Slider is used for an element body
- A body which shows elastic deformation until yield point
  - Perfectly plastic body



Slider?





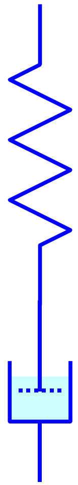
$$\text{Yield strength} = \text{Maximum value of frictional force} = \mu mg$$

where μ is the friction coefficient

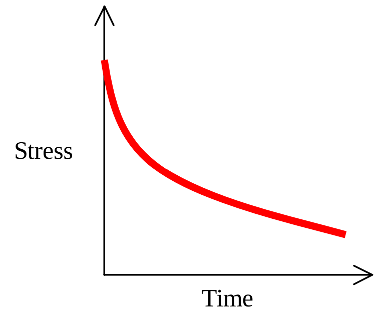
6.2.2 Mechanical model

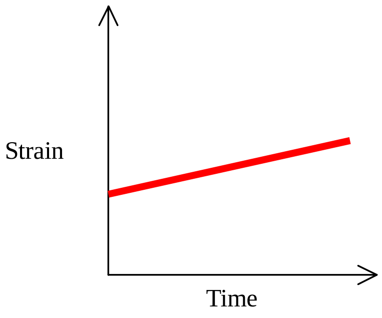
Maxwell body

- One of the models which shows visco-elasticity
  - Stress relaxation occurs when stress is kept constant
  - Strain increases under constant stress



- Sticky candy etc.





$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = E\frac{\mathrm{d}\varepsilon_{\mathrm{s}}}{\mathrm{d}t}$$

Ttal strain ε is

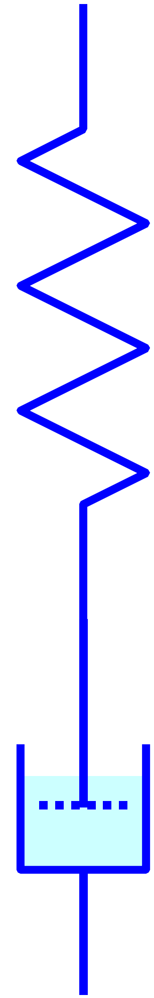
$$\varepsilon = \varepsilon_{\mathrm{s}} + \varepsilon_{\mathrm{d}}$$

Therefore

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{\mathrm{d}\varepsilon_{\mathrm{s}}}{\mathrm{d}t} + \frac{\mathrm{d}\varepsilon_{\mathrm{d}}}{\mathrm{d}t}$$

$$= \frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{\sigma}{\eta}$$

(Maxwell equation)



$$\sigma = E\varepsilon_s$$

$$\sigma = \eta\dot{\varepsilon}_{\mathrm{d}}$$

Constant strain

Assume that strain is kept constant in

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{\sigma}{\eta}$$

Then,

$$0 = \frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{\sigma}{\eta}$$

$$\frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}t} = -\frac{\sigma}{\eta}$$

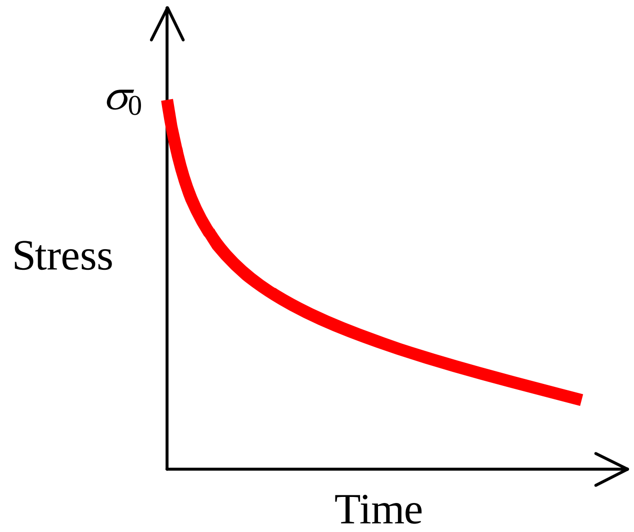
$$\frac{1}{E}\frac{\mathrm{d}\sigma}{\sigma} = -\frac{1}{\eta}\mathrm{d}t$$

$$\frac{\ln\sigma}{E} = -\frac{t}{\eta} + C_1$$

$$\sigma = \exp\left(-\frac{Et}{\eta} + C_1\right) = \exp(C_1)\exp\left(-\frac{Et}{\eta}\right)$$

Assume that $\sigma = \sigma_0$ at $t = 0$,

$$\sigma = \sigma_0 \exp\left(-\frac{Et}{\eta}\right)$$



Stress relaxation

Constant stress

Assume constant stress in

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{\sigma}{\eta}$$

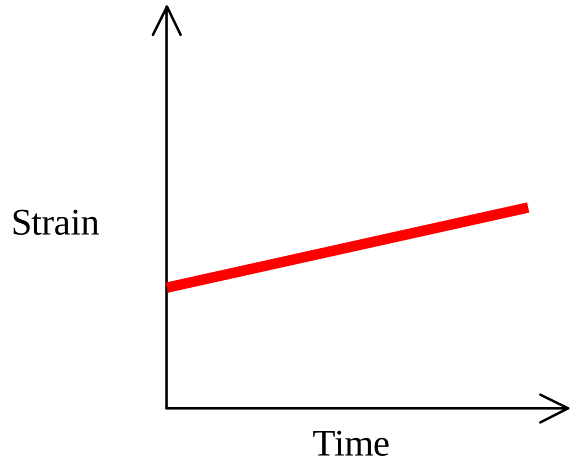
Then,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{\sigma}{\eta}$$

$$\varepsilon = \frac{\sigma}{\eta}t + C_1$$

Since $\varepsilon = \frac{\sigma}{E}$ at $t = 0$,

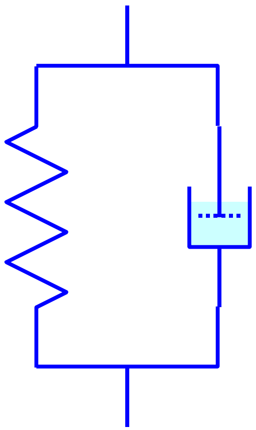
$$\varepsilon = \frac{\sigma}{E} + \frac{\sigma}{\eta}t$$



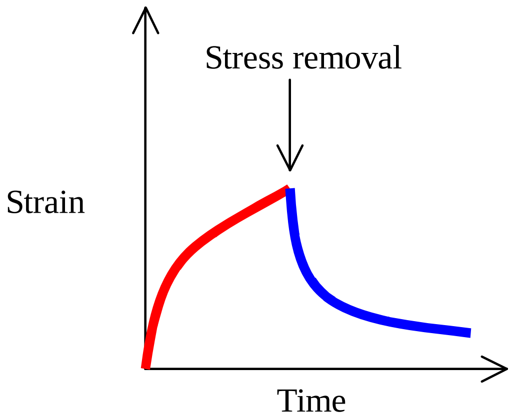
Viscous deformation

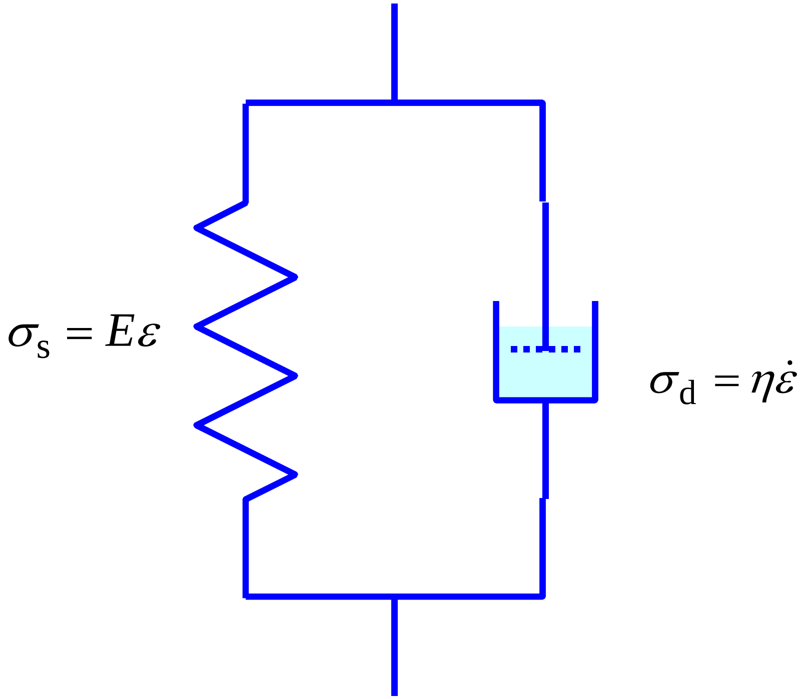
Voigt body (Kelvin body)

- One of the models for visco-elastic body
  - Strain increases with decreasing strain rate under constant stress
  - Strain decreases with decreasing strain rate when stress is removed



- Nylon etc.





Total stress σ is

$$\sigma = \sigma_s + \sigma_d$$

Therefore,

$$\sigma = E\varepsilon + \eta\frac{d\varepsilon}{dt}$$

(Voigt or Kelvin equation)

In the case where stress is kept constant between $t = 0$ and $t = t_1$

$$\sigma = E\varepsilon + \eta \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}$$

$$\eta \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -E\varepsilon + \sigma$$

$$\frac{\mathrm{d}\varepsilon}{-E\varepsilon + \sigma} = \frac{1}{\eta}\mathrm{d}t$$

$$-\frac{\ln(-E\varepsilon + \sigma)}{E} = \frac{t}{\eta} + C_1$$

$$\ln(-E\varepsilon + \sigma) = -E\frac{t}{\eta} + C_2$$

$$-E\varepsilon + \sigma = \exp C_2 \exp\left(-E\frac{t}{\eta}\right)$$

$$-E\varepsilon = \exp C_2 \exp\left(-E\frac{t}{\eta}\right) - \sigma$$

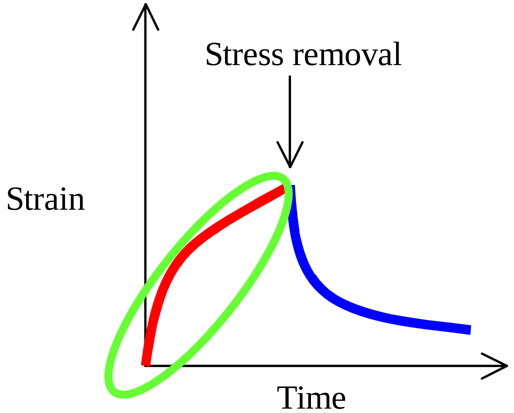
$$\varepsilon = C_3 \exp\left(-E\frac{t}{\eta}\right) + \frac{\sigma}{E}$$

Assuming that $\varepsilon = 0$ when $t = 0$,

$$\varepsilon = -\frac{\sigma}{E}\exp\left(-E\frac{t}{\eta}\right) + \frac{\sigma}{E}$$

$$\varepsilon = \frac{\sigma}{E}\left\{1 - \exp\left(-E\frac{t}{\eta}\right)\right\}$$

retarded elasticity



Then, unloaded

Substituting $\sigma = 0$, $t = t_1$ and strain at $t = t_1$

$$\varepsilon = C_3 \exp\left(-E\frac{t}{\eta}\right) + \frac{\sigma}{E}$$

into

$$\varepsilon = \frac{\sigma}{E}\left\{1 - \exp\left(-E\frac{t_1}{\eta}\right)\right\}$$

we get

$$\frac{\sigma}{E}\left\{1 - \exp\left(-E\frac{t_1}{\eta}\right)\right\} = C_3 \exp\left(-E\frac{t_1}{\eta}\right)$$

Therfore

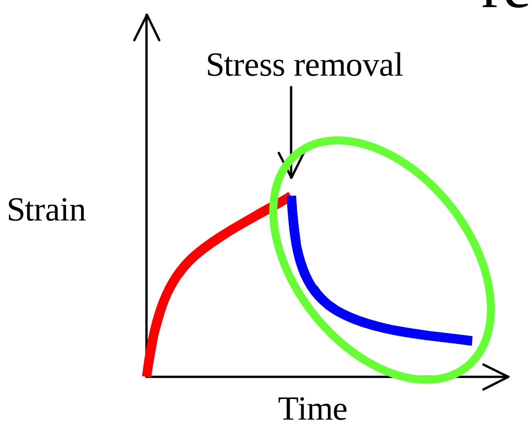
$$C_3 = \frac{\frac{\sigma}{E}\left\{1 - \exp\left(-E\frac{t_1}{\eta}\right)\right\}}{\exp\left(-E\frac{t_1}{\eta}\right)}$$

Hence,

$$\varepsilon = \frac{\frac{\sigma}{E}\left\{1 - \exp\left(-E\frac{t_1}{\eta}\right)\right\}}{\exp\left(-E\frac{t_1}{\eta}\right)} \exp\left(-E\frac{t}{\eta}\right)$$

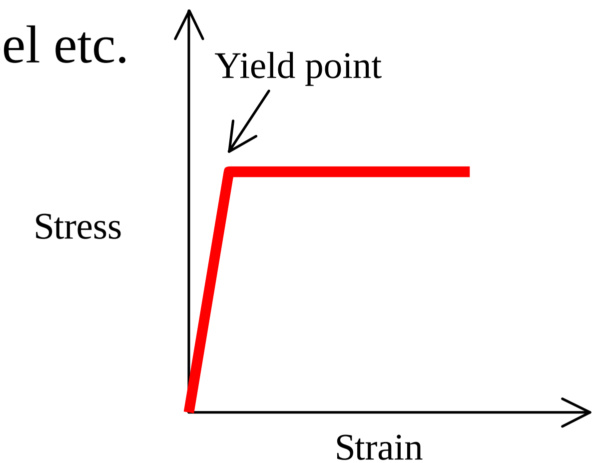
$$\varepsilon = \frac{\sigma}{E}\left\{1 - \exp\left(-E\frac{t_1}{\eta}\right)\right\} \exp\left(-E\frac{t - t_1}{\eta}\right)$$

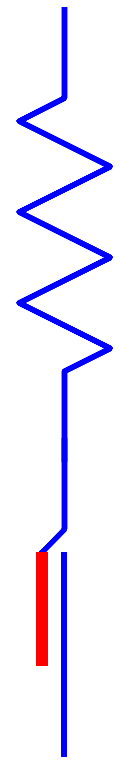
retarded elasticity



St. Venant body

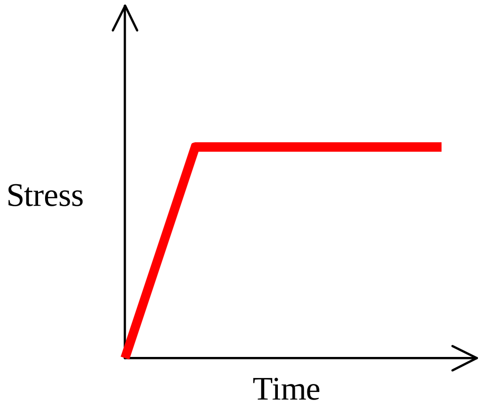
- Elasto-plastic body
- Elastic deformation until yield point
- Plastic deformation after yield point
  - Tooth paste, mild steel etc.

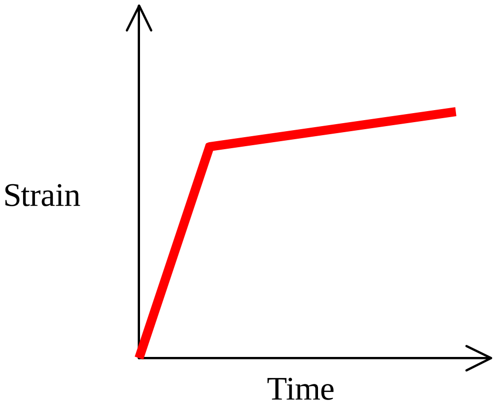


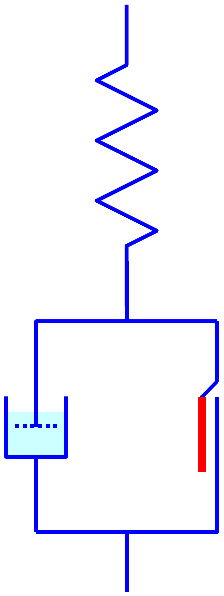


Bingham body

- A model for Visco-elasto-plastic body
- Elastic deformation until yield point
- Viscous deformation after yield point

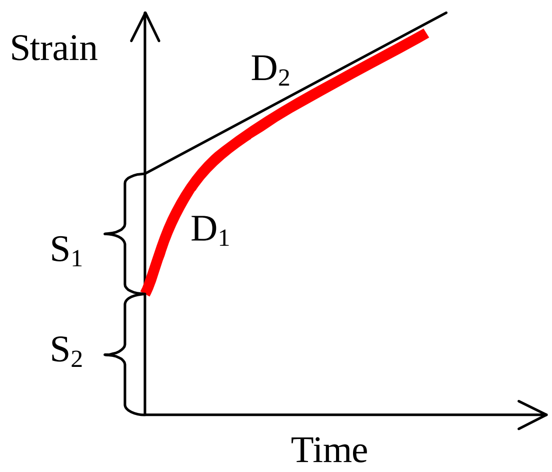


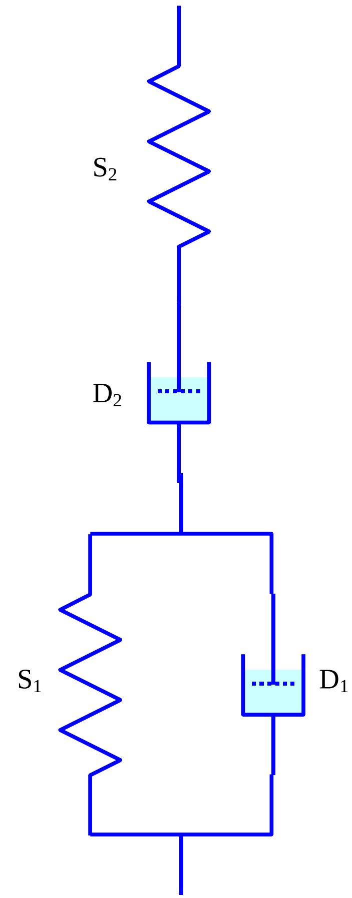




Mechanical model for creep deformation of rock

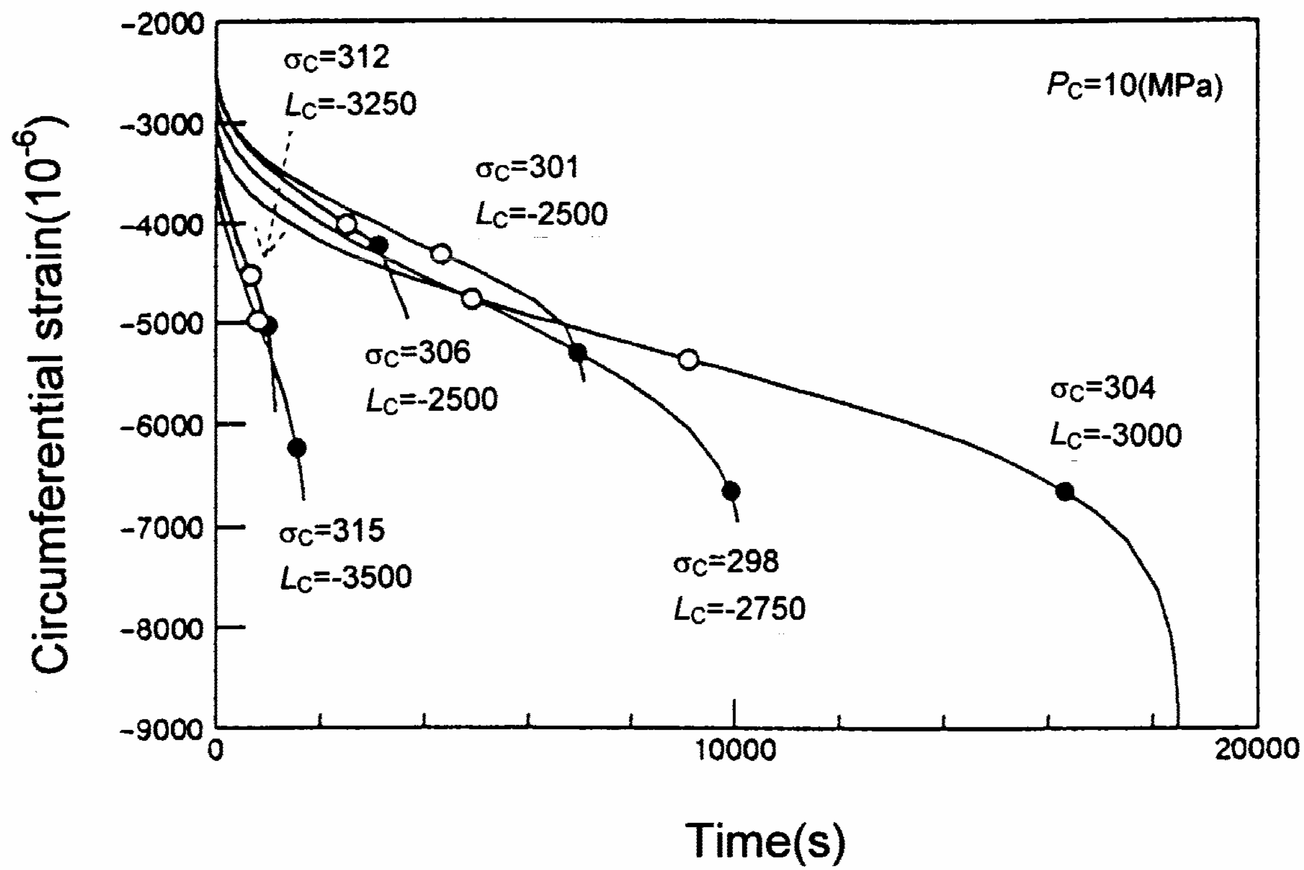
- Burgers model for primary and steady creep
- Tertiary creep can not be easily represented.



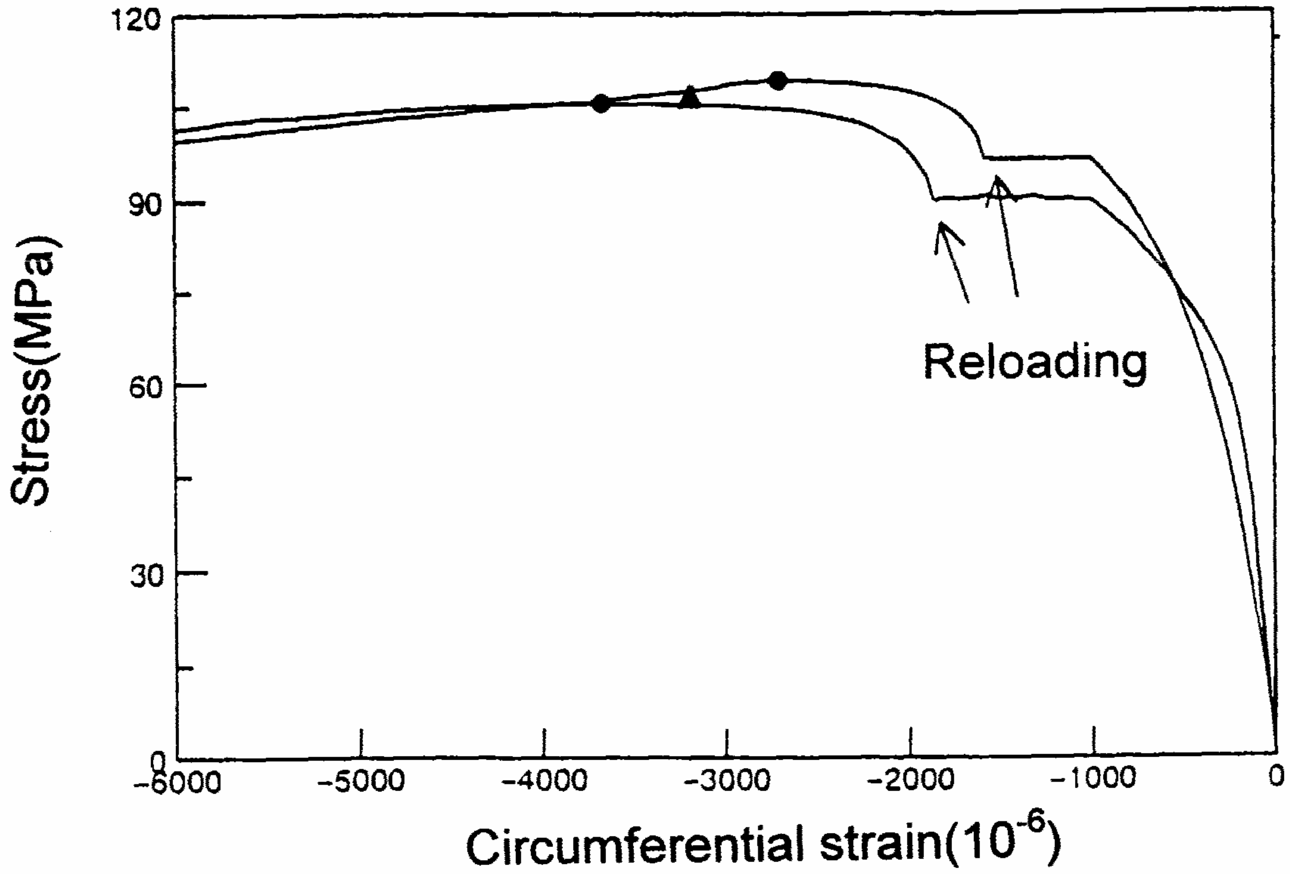


6.3 Example of creep test

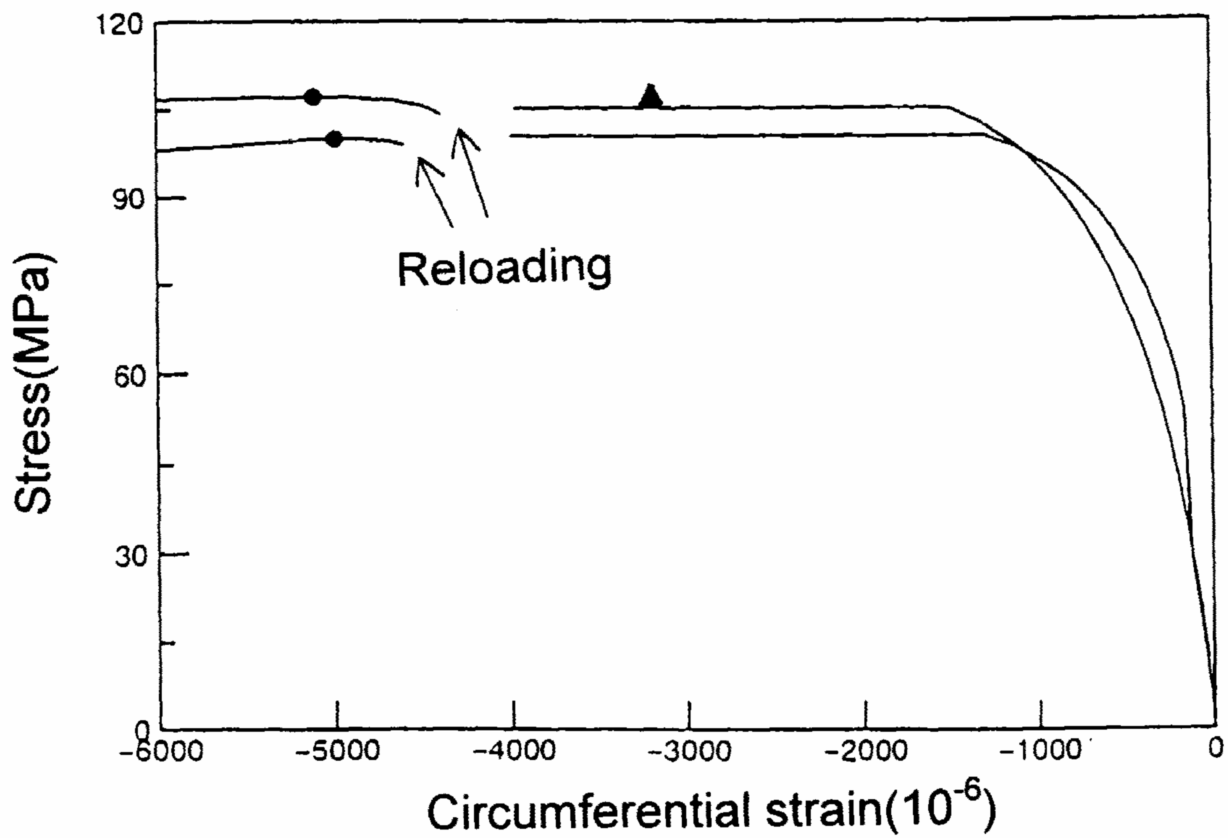
Creep behavior in circumferential strain



Damage of rock



The same strength can be attained when the specimen is reloaded before minimum circumferential strain rate is attained.



Peak stress is less than that in constant strain rate test when the specimen is reloaded after minimum circumferential strain rate is attained.

6.4 Mechanism of creep

Main causes of creep

- Change in distance between mineral particles
  - Migration of pore water, shear displacement, detachment etc.
- Deformation of mineral particles
  - Pressure solution, diffusion, dislocation etc.
- Subcritical crack growth
  - Stress corrosion etc.

Migration of pore water

- Finite velocity in migration of pore water due to pressure gradient by specimen deformation.
  - Hydraulic conductivity
  - Viscosity
    - Temperature
- The primary creep can be explained by homogenization of pore pressure.
- Effect of temperature can be explained.

Pressure solution

- Quartz etc. can be dissolved in pore water by high stress concentration at contact points.
- The solution migrates by grain boundary diffusion etc.
- Quartz etc. are precipitated at low stress-region.
  - Primary creep can be explained by the increase of contacted area.
  - Effects of temperature, pH etc. can be explained?

Diffusion creep

- Homogenization of density
  - Voids: from a grain boundary under tension to that under compression
  - Atoms: from a grain boundary under compression to that under tension

Let L be the grain size. The density gradient is $\frac{\sigma}{L}$,

In the diffusion inside the grain, displacement rate is

proportional to density gradient, $C\frac{\sigma}{L}$,

strain rate is, dividing the disdplacement rate by the grain size,

$$\dot{\varepsilon} = C\frac{\sigma}{L^2}$$

In the grain boundary diffusion, cross sectional area is WL

where W is the thickness of grain boundary. The area is $\frac{WL}{L^2}$ times

the grain boundary diffusion case. Therefore,

$$\dot{\varepsilon} = \frac{C\sigma W}{L^3}$$

- Strain rate in diffusion creep
  - Inversely proportional to square or cube of the grain size
  - Proportional to creep stress

- Diffusion creep can be observed for metals and ceramics under low creep stresses.
- Existence of diffusion creep in minerals and rocks has not been proven.

Dislocation creep

Edge dislocation

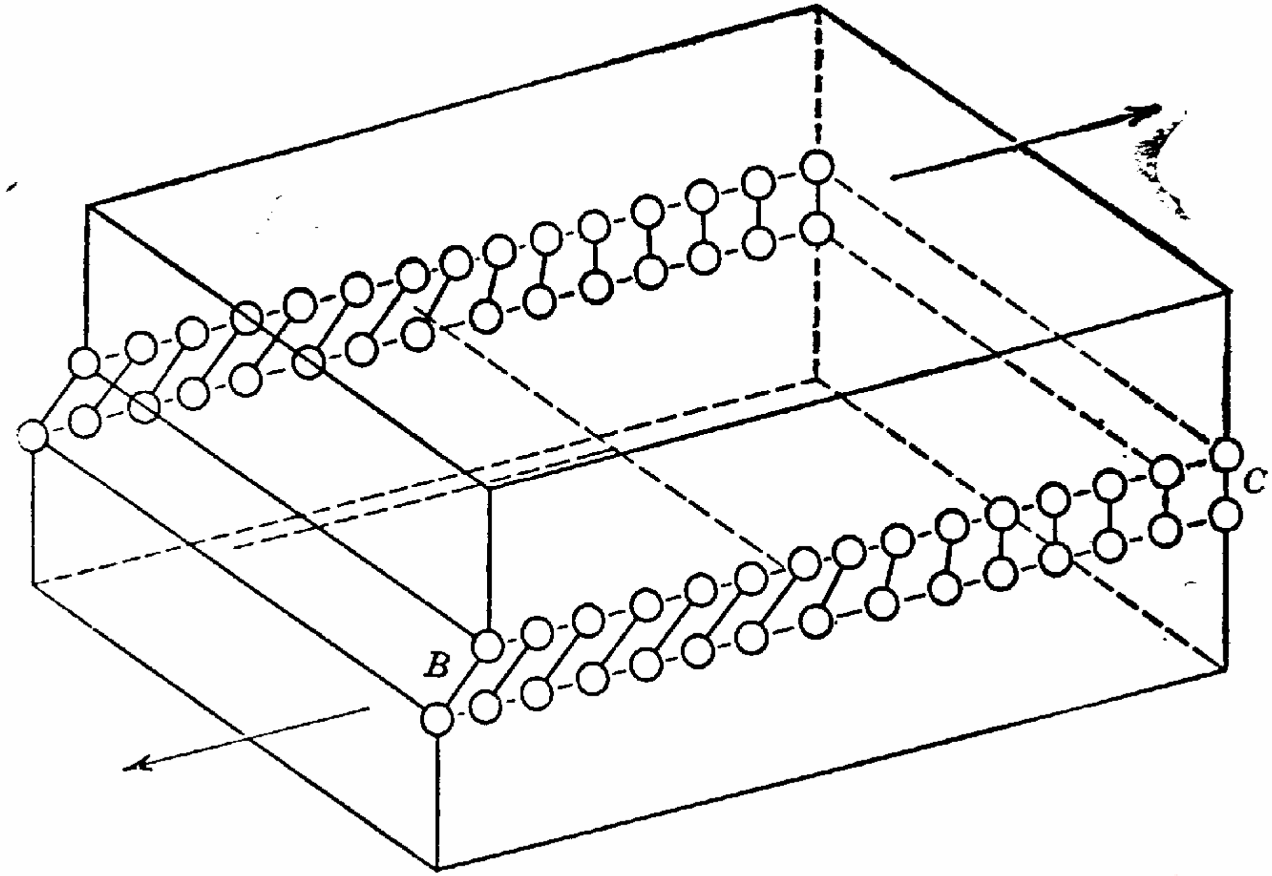
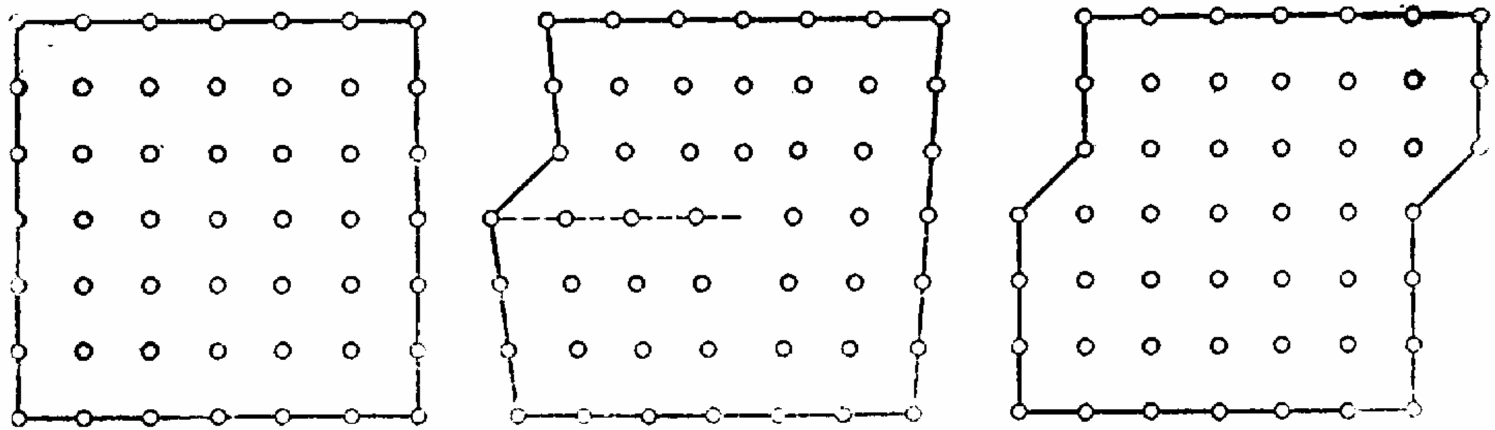


図 11-2 スリップによって生じた刃状転位*

制限資料

飯田修一(1965)、技術者のための固体物性、第2版、丸善、p. 228図11-2


正 の 転 位

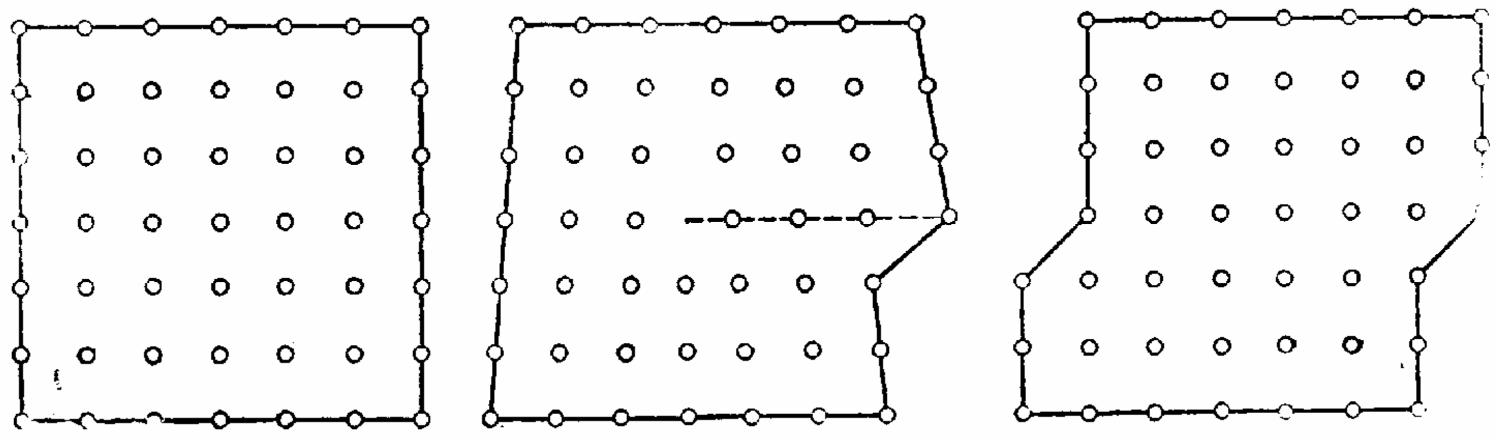

負 の 転 位

図 11-3　スリップを作る転位の動きの説明図**

制限資料

飯田修一(1965)、技術者のための固体物性、第2版、丸善、p. 228図11-3

Screw dislocation

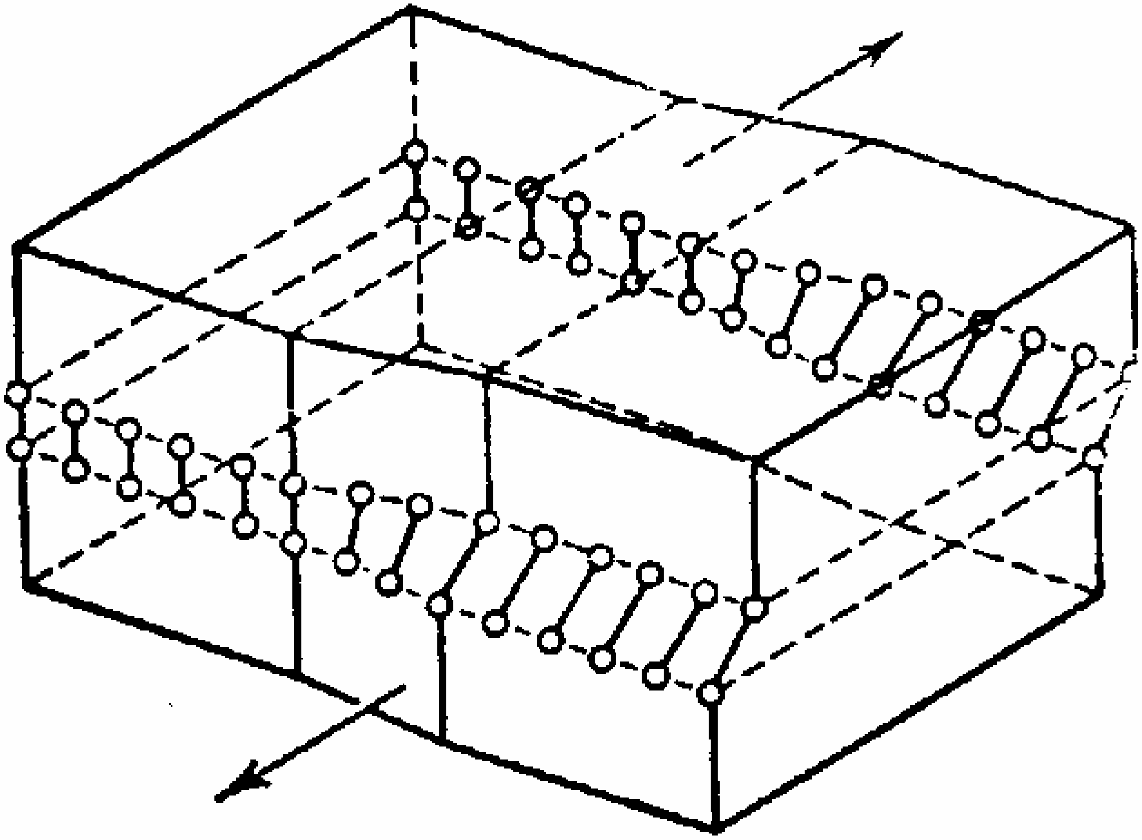


図 11-4 ラセン転位の構造*

制限資料

飯田修一(1965)、技術者のための固体物性、第2版、丸善、p. 229図11-4

Strain rate by dislocation is proportional to density and velocity of dislocation.

$$\dot{\varepsilon} = C' \rho v$$

Density and valocity of dislocation is proportional to square of stress and stress, respectively.

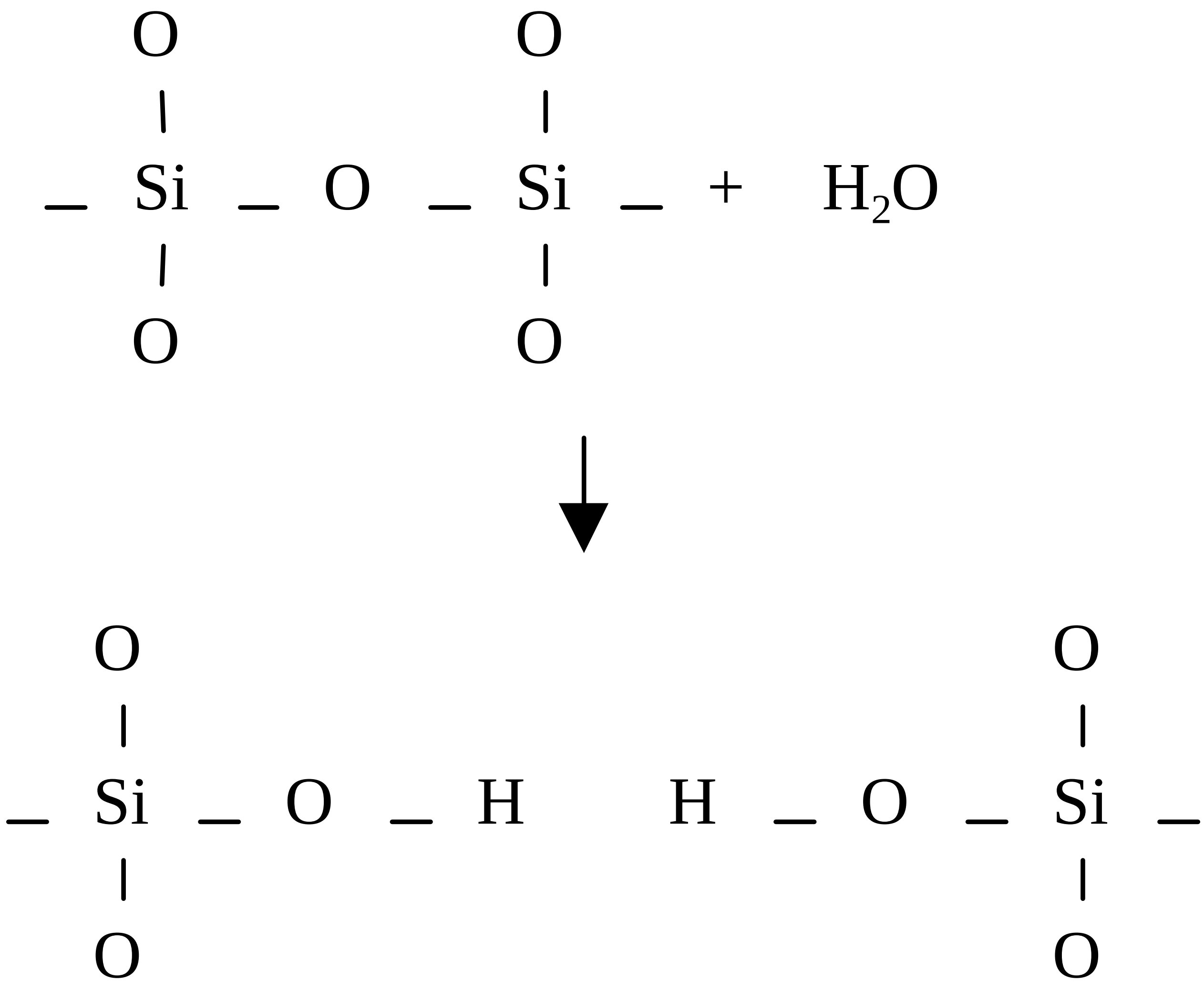
$$\dot{\varepsilon} = C'' \sigma^3$$

- Strain rate by dislocation creep is proportional to cube of stress.
- Creep strain rate of minerals and rocks under high temperature sometimes proportional to the cube of stress.
- Existence of dislocation has not proven

Stress corrosion

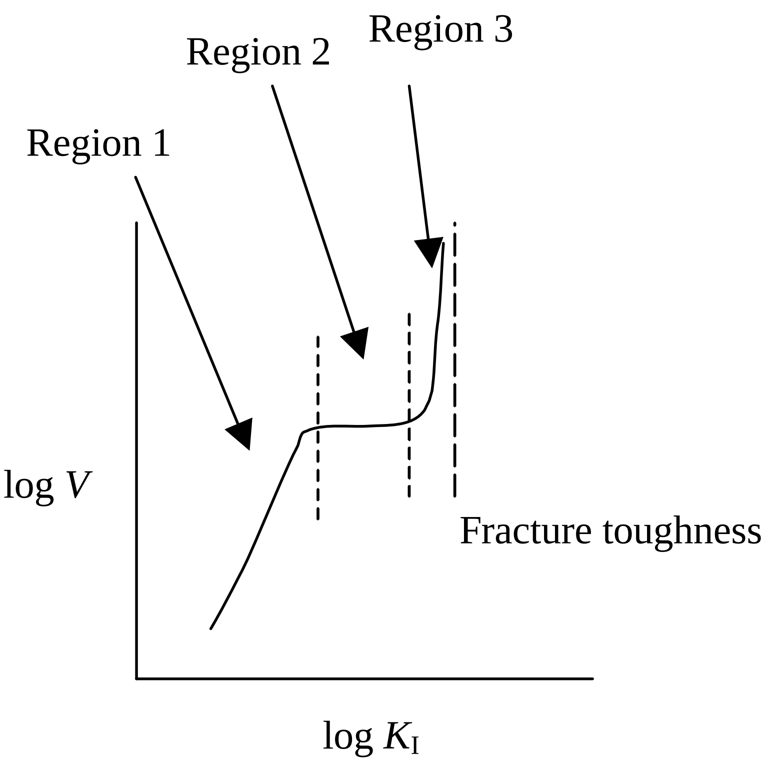
- Corrosion of elongated and weakened atomic bonds by surround substances.

- Substances which cause corrosion
  - H_2O for SiO_2
  - CH_3CN for MgF_2


O O
Si O Si + H2O
O O
O O
Si O H H O Si
O O

Crack velocity and stress intensity factor

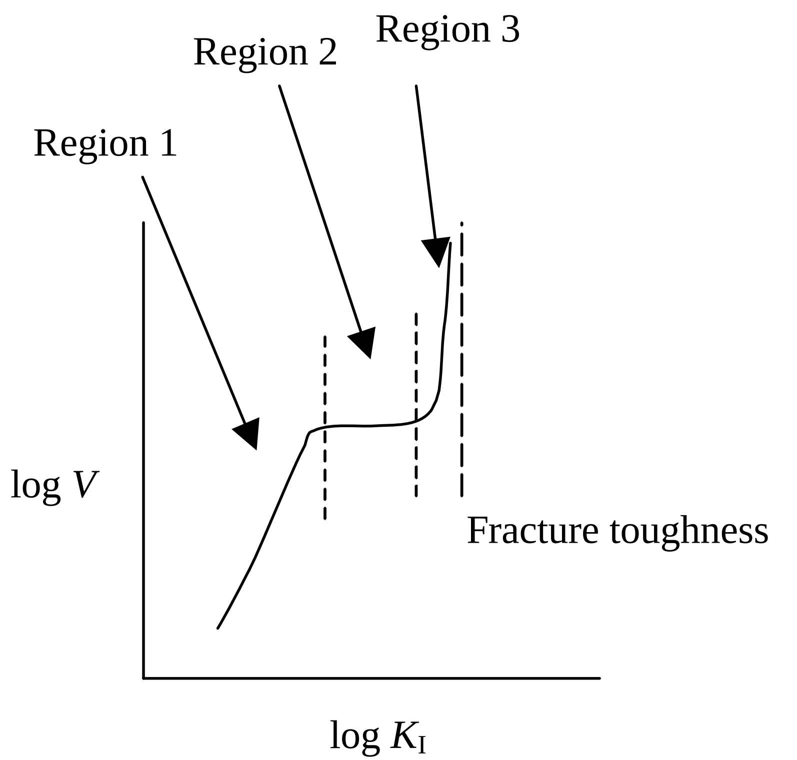
- Region 1
  - Stress corrosion rate increases with stress intensity factor→crack velocity increases
- Region 2
  - Crack velocity is almost constant since diffusion velocity of the corrosion material determines the corrosion rate.
  - This region would not be observed clearly for inhomogeneous materials.
- Region 3
  - Failure due to mechanical elongation of atomic distance.



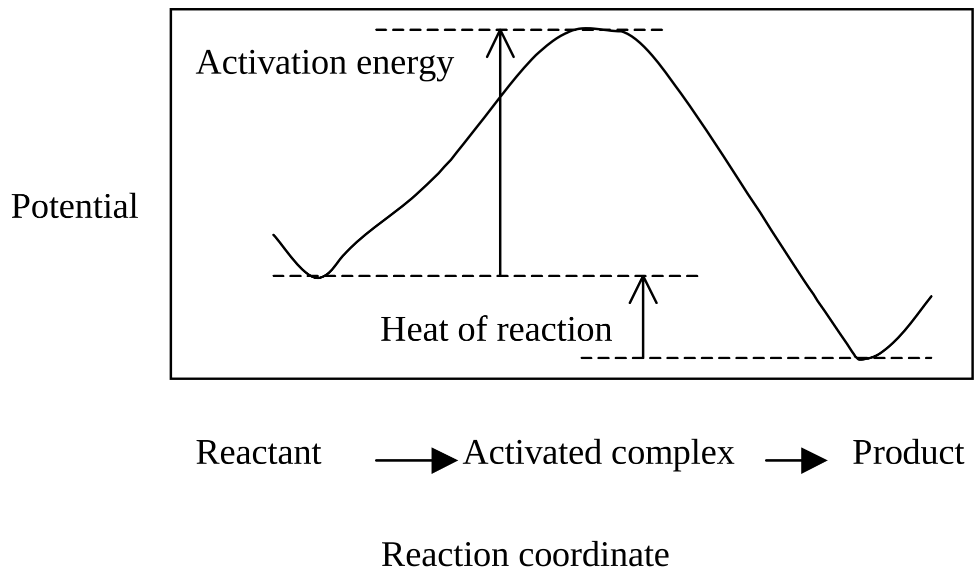
Upper limit of crack velocity

- 0.38 times P-wave velocity due to the limit of energy supply rate.

- The upper limit of velocity of fracture propagation in fault earthquake is velocity of Reyleigh wave (0.88～0.96 times S-wave velocity)
  - Difference in new fracture and propagation of slip along already fractured fault.



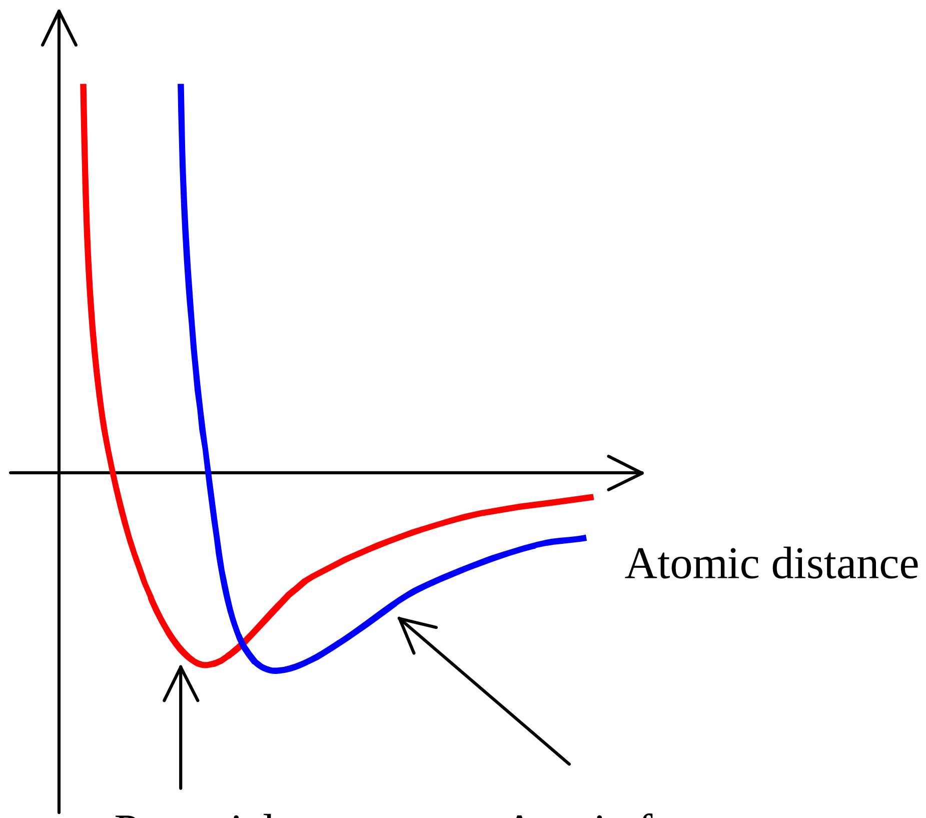
Activation energy



- Activation energy has to be exceeded for a reaction
  - The higher the energy, the higher the probability to exceed the activation energy
  - Increase in reaction rate - increase of crack velocity
- Number of reaction molecules and the density of corrosion material have a positive correlation in a certain density range

How to increase energy?

- Raising temperature (enthalpy)
- Elongation (potential)



$$V = V_0 a(\mathrm{H_2O}) \exp\left(\frac{-E_{\mathrm{act}} + \alpha K_{\mathrm{I}}}{RT} \right)$$

$$\alpha = \frac{2V^*}{\sqrt{\pi r_c}}$$

Crack velocity in Region 1

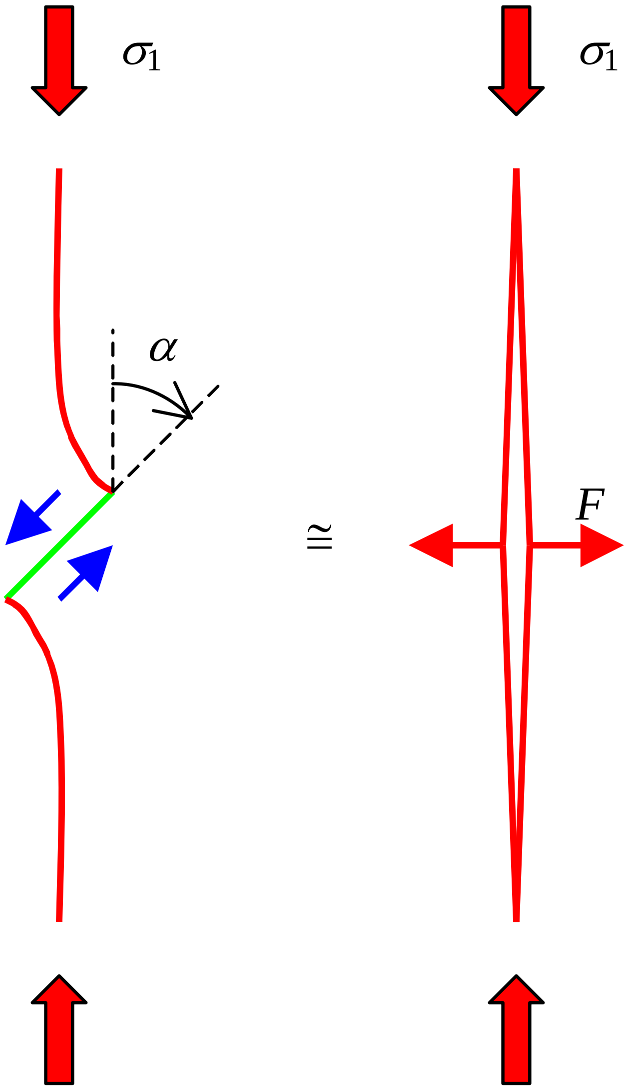
- V_0 (m/s) is a constant depending material and conditions
- $a(\mathrm{H_2O})$ is the activity of water that is nearly equal to the relative humidity
- E_{act} (J/mol) is the activation energy per 1 mol
- K_{I} is the stress intensity factor for Mode I ($\mathrm{Pa \cdot m^{0.5}}$)
- R is the gas constant (8.3144 J $\mathrm{K^{-1}}$ $\mathrm{mol^{-1}}$)
- T is the absolute temperature (K)
- V^* is the volume of activation
- r_{c} is the radius of curvature at crack tip
- Value in the parenthesis is below zero.
- V_0 is the value of V when the value in the parenthesis is zero, namely, infinite high temperature or $\alpha K_{\mathrm{I}} = E_{\mathrm{act}}$
- Crack velocity decreases with decrease od temparature from infinite high temperature or decrease of stress intensity from E_{act}/α

- Q: Crack velocity of a rock under a small stress is 1 mm/s and 10 mm/s at 30°C and 200°C, respectively. Estimate crack velocity at 0°C.
- A: 0.497 mm/s

Explanation of primary creep based on stress corrosion

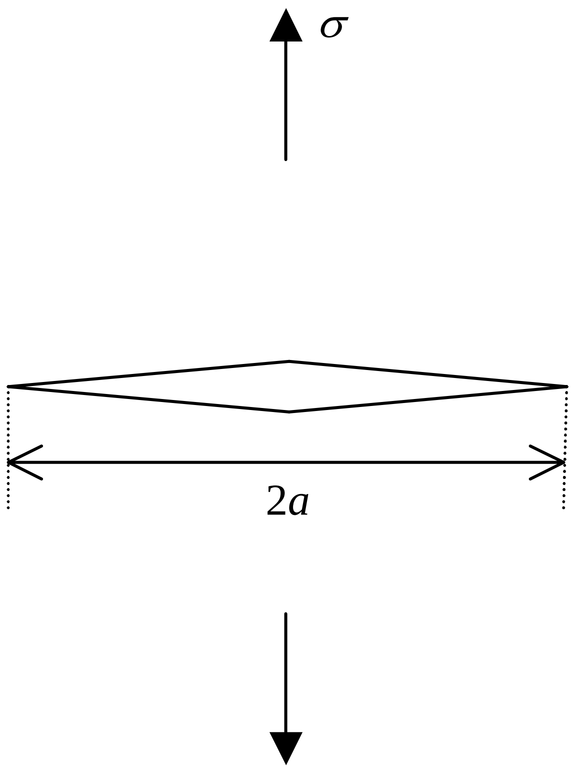
$$K_{\mathrm{I}} = \frac{F}{\sqrt{\pi l}}$$

- The longer the crack grows,
  - The smaller the stress intensity factor becomes
  - Crack velocity decreases
  - Strain rate decreases



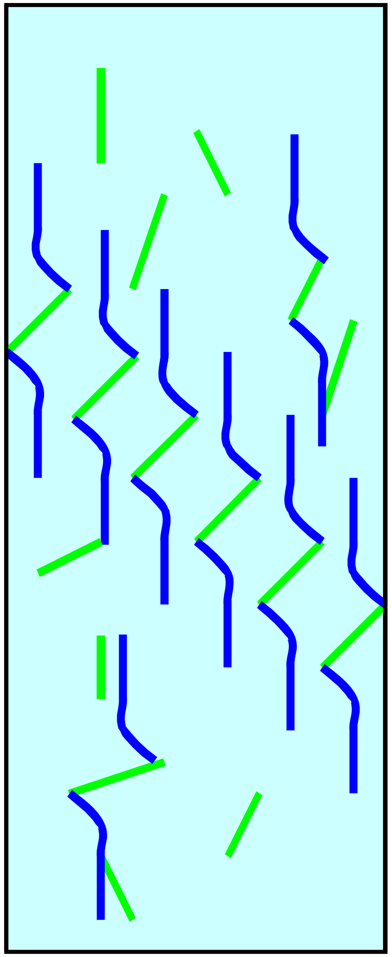
$$K_{\mathrm{I}} = \sigma\sqrt{\pi a}$$

- When the crack grows under tensile stress, stress intensity increases.
  - Primary creep shouldn't appear but was observed.



Explanation of tertiary creep based on stress corrosion

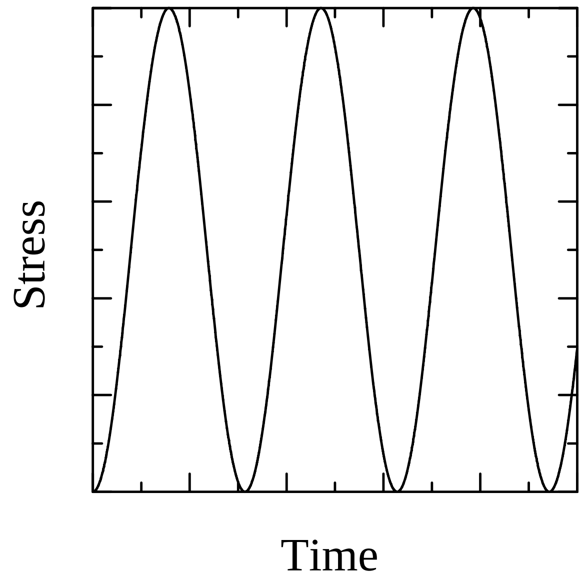
- When crack density exceeds a certain limit, stress intensity factor increases due to interferences between the cracks
  - Crack velocity increases.
  - Strain rate increases.



6.5 Fatigue failure

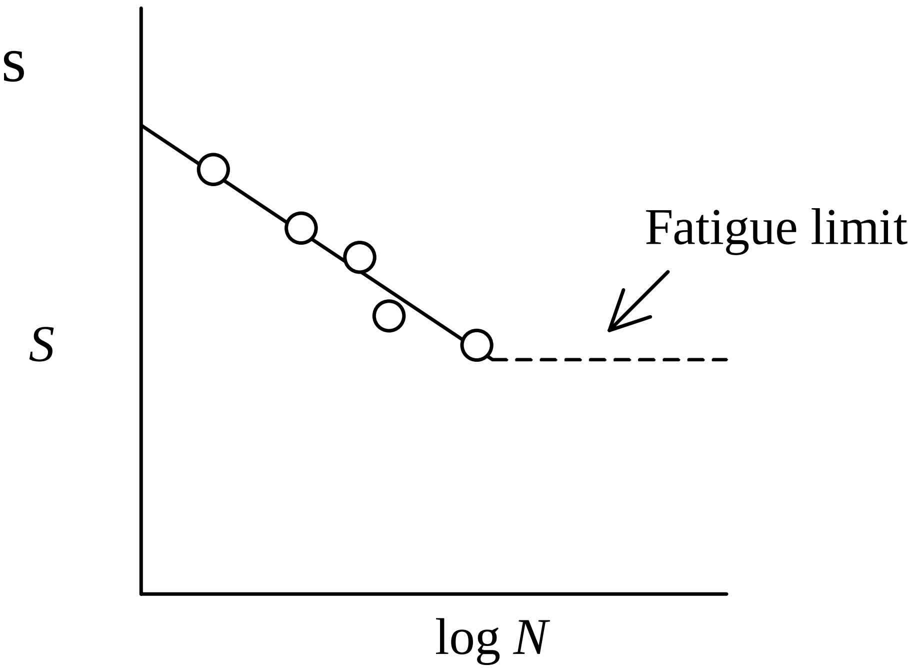
Fatigue failure?

- Failure due to cyclic loading
  - Fatigue failures of metallic parts is causes of many accidents of airplanes, nuclear power plants.



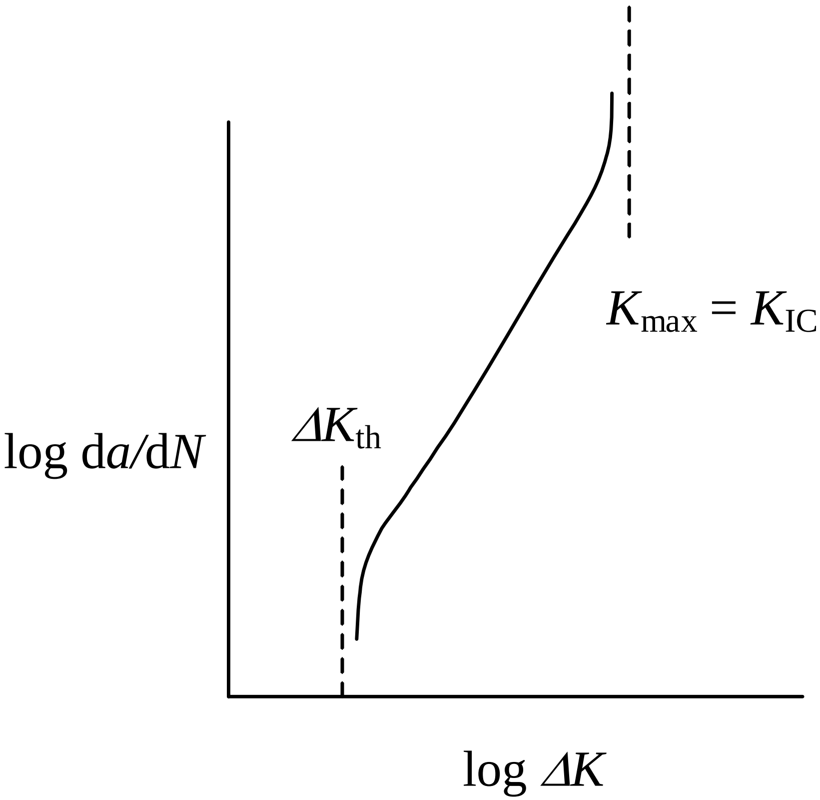
S-*N* curve

- *S* :stress amplitude
- *N* :number of cycles

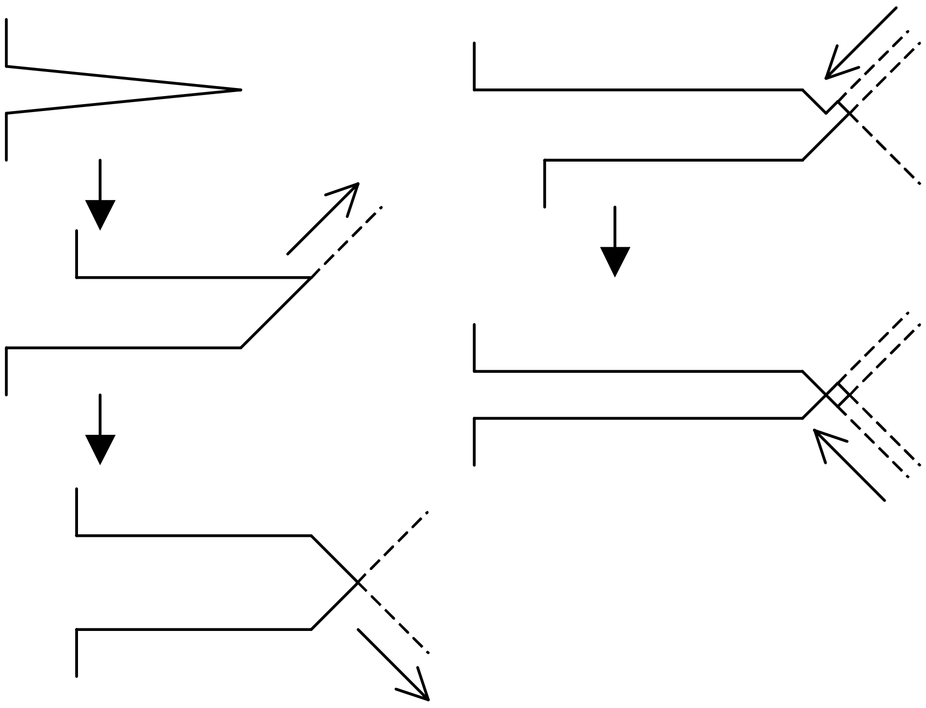


Characteristics of fatigue crack growth

- da/dN :increment of crack length in a loading cycle
- ΔK_{th}:lower limit of stress intensity factor amplitude
  - Crack never grow under stress intensity factor below the limit.

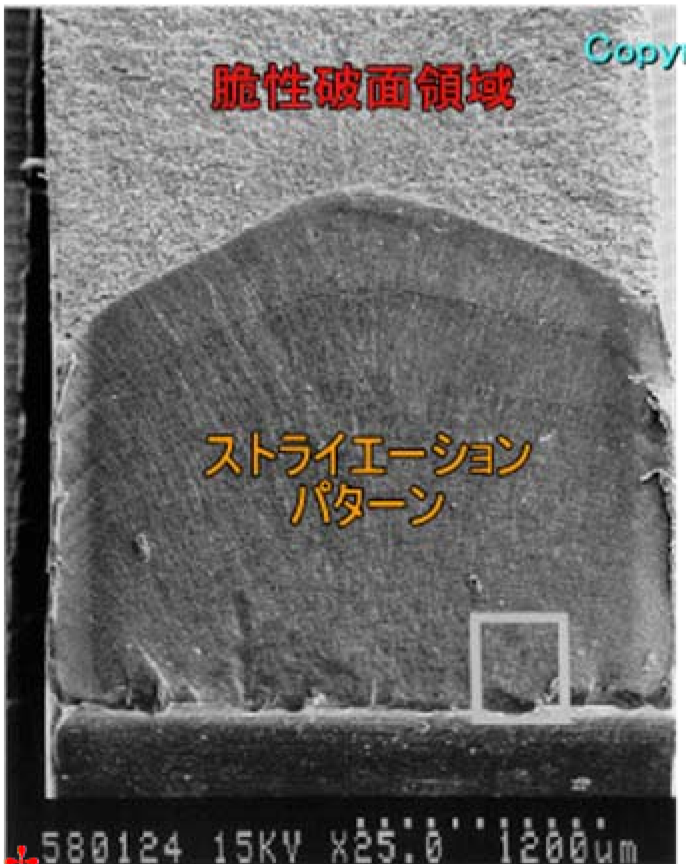


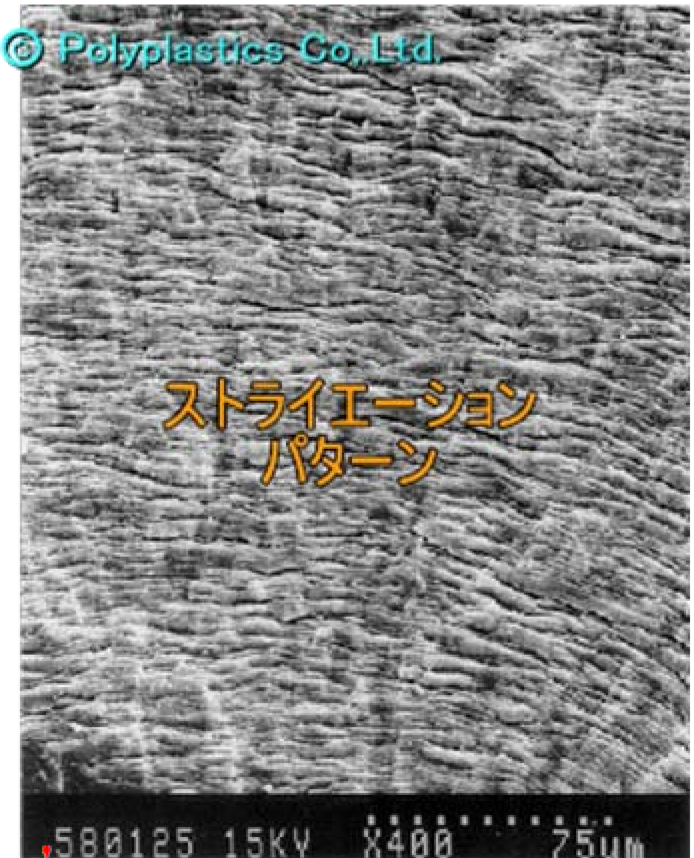
Mechanism of fatigue



Loading process Unloading process

Characteristics of fatigue crack





制限資料

制限資料

Striation pattern

PLAMOS web-site内
「http://www.plamos.com/html/support/s_10_03_04.html」
に掲載してある電顕写真

- Fatigue crack grows in plastic regions which were created by the crack itself.
- There are many oxidized or corroded materials at crack surfaces.
- They prevent crack growth under very small stress intensity amplitude

Fatigue crack of rock

- Time to failure becomes maximum at a certain stress amplitude.