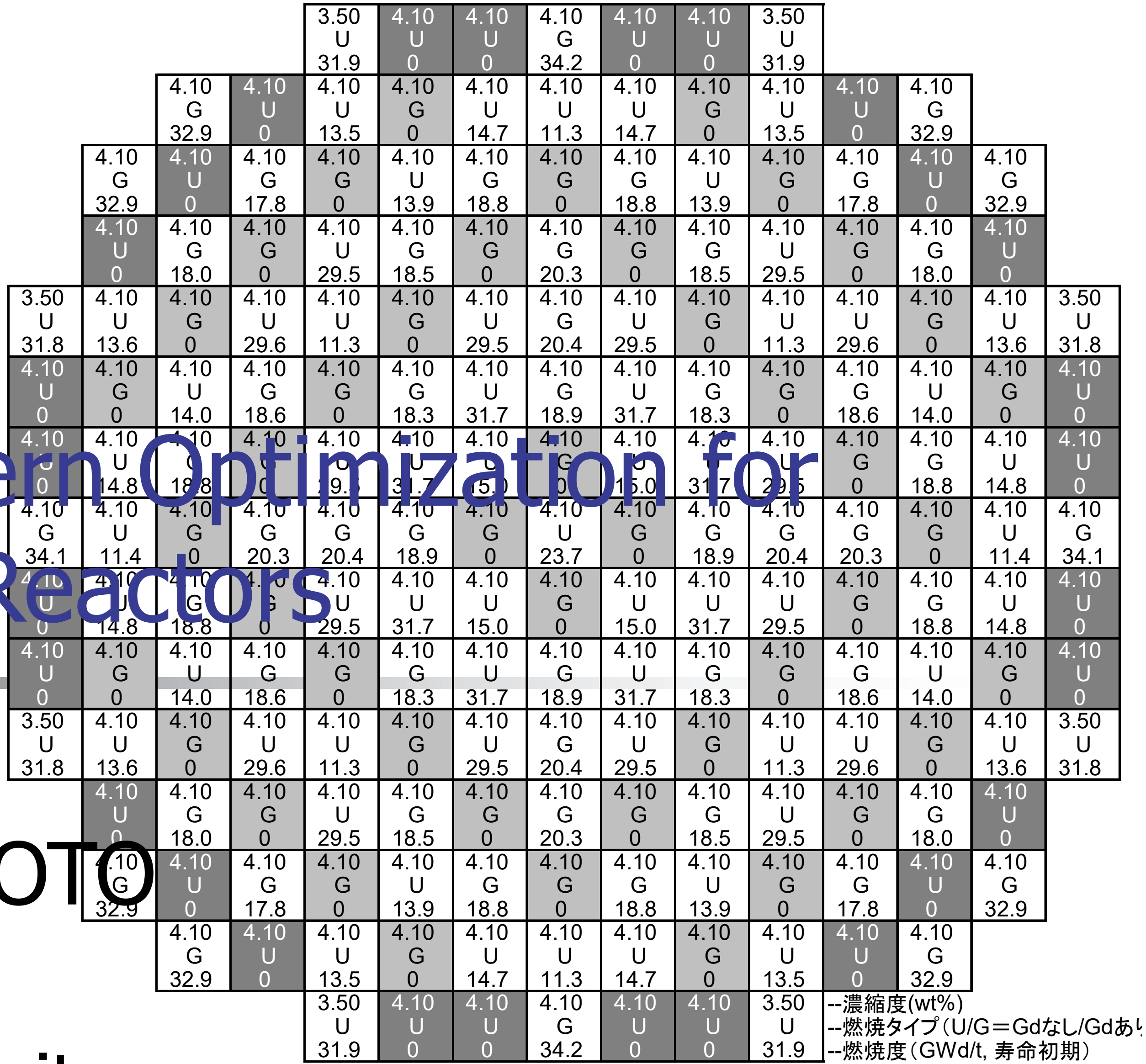
Loading Pattern Optimization for Commercial Reactors



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Contents of this lecture

- In-core fuel management tasks in PWR
- Basic theory of in-core fuel management
  - Linear reactivity model
  - Cycle length, discharge burnup and batch size
  - Analysis of power sharing
- Loading pattern optimizations
  - Overview
  - Objectives and constraints
  - Simulated annealing/Genetic algorithms/Tabu-search/Heuristics
- Summary

In-core fuel management tasks in PWR

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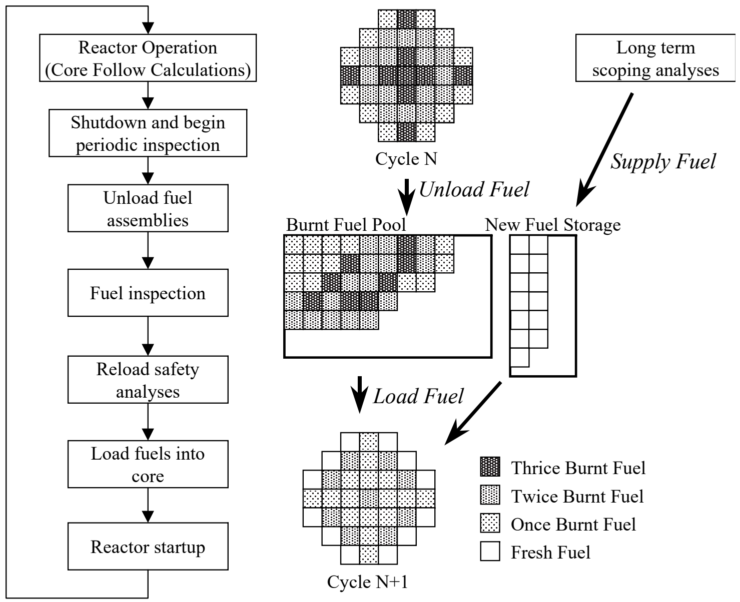


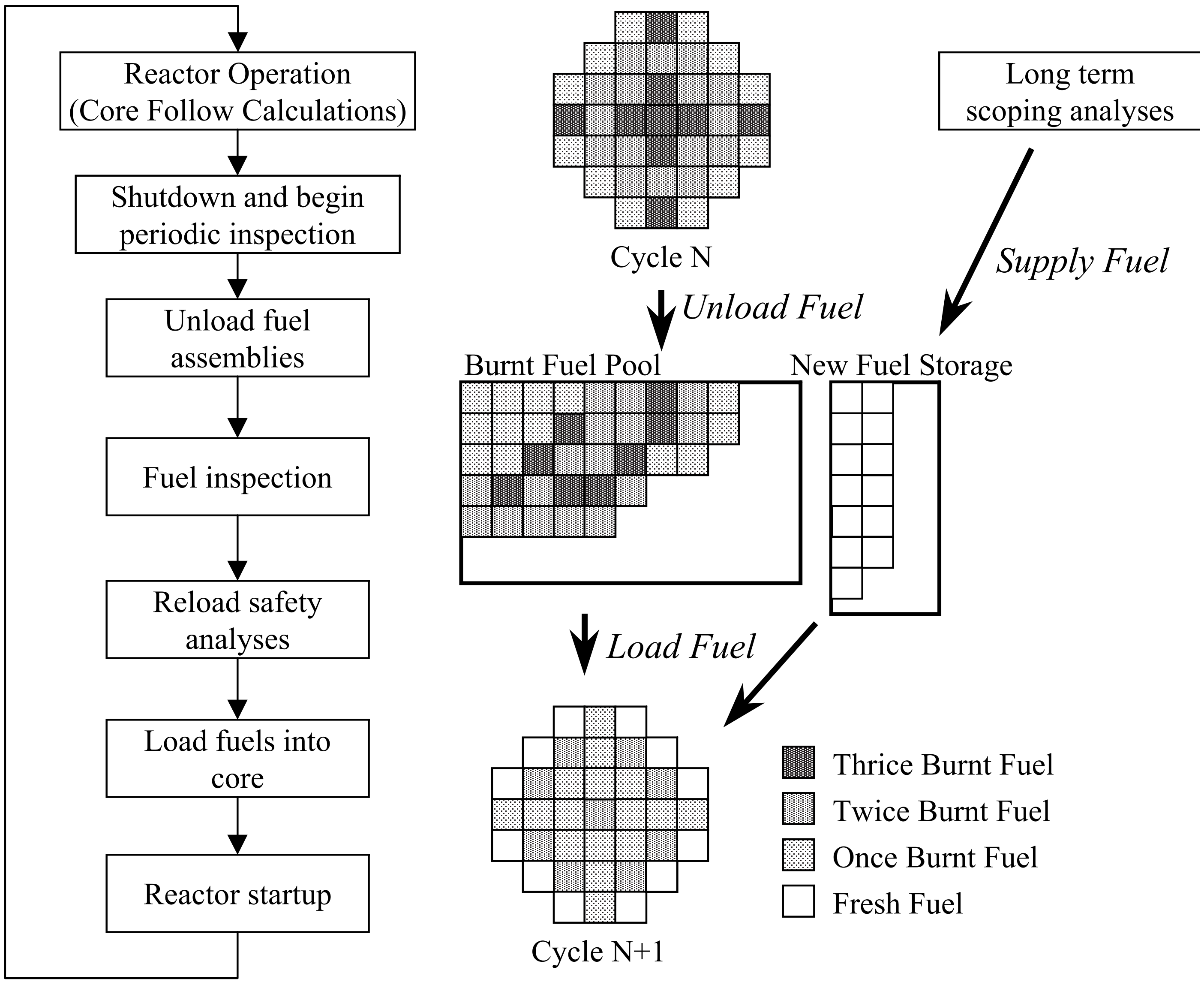
Outline of in-core fuel management tasks

- The objective of in-core fuel management
  - Establish a strategy for effective utilization of fuels while maintaining core safety
- In-core fuel management tasks are usually divided into three parts
  - Long term scoping analyses
  - Reload core analyses
  - Startup/core follow analyses

Outline of in-core fuel management tasks

- Long term scoping analyses
  - Establish a plan for fuel order during next several years (cycles)
  - Simplified core calculation models (0D-2D) are used
- Reload core analyses
  - Finalize feed/reload assemblies, develop a final loading pattern and perform safety analysis of the reload core
  - Detailed core calculation models (3D) are required for accurate prediction of core performance
- Startup/core follow analyses
  - Startup testing (startup reactor physics test)
  - Core monitoring to confirm status of in-core fuel assemblies

Typical flow of in-core fuel management in Japan



Comparison of in-core fuel management activities

Type	Operating plan	Fuel stock	Analyses scope period(years) 0 1 2 3 4 5 10 15	
Long term fuel mng. strategy	free	free	Optimum fuel design study etc.	
Long term fuel purchase plan	fix	free	Analyses of fuel/BP inventory	
Short term fuel management	fix	fix	Analyses of effective fuel management methods	Rough study of LP
Next cycle detail core design	fix	fix	Design fuel loading pattern	Detail study of LP

Long term scoping analyses: Overview

- Establish a plan for fuel order during next several years (cycles)
  - Lead time for fuel fabrication is long
- Outline of fuel management plan is fixed by long term scoping analyses; significant impact on fuel cycle cost
- Generally simplified core calculation models (0D-2D) are used due to complicated analyses sequences and required calculation accuracy

Reload core analyses: Objectives

- Determine a loading pattern of next cycle subject to:
  - Core safety during operation
  - Cycle length requirement
  - Fresh fuel stock
  - Fuel failure
  - Lead test assembly
  - Other limitations
- Economical aspect is also important
  - The core performance almost rely on a loading pattern especially in PWR

Reload core analyses: Typical time schedule

Days before startup	Actions / Events
70	Scoping analyses of next cycle
40	Reactor shutdown
30	Fuel inspection
25	Reload core analyses
20	Fuel shuffling
14	Startup test analyses
	Documents for reactor operation
0	Reactor startup testing
	Operation approval from licensing authority
-2	Reach to full power operation
every one month	In-core power measurement by MD

continue to next cycle...

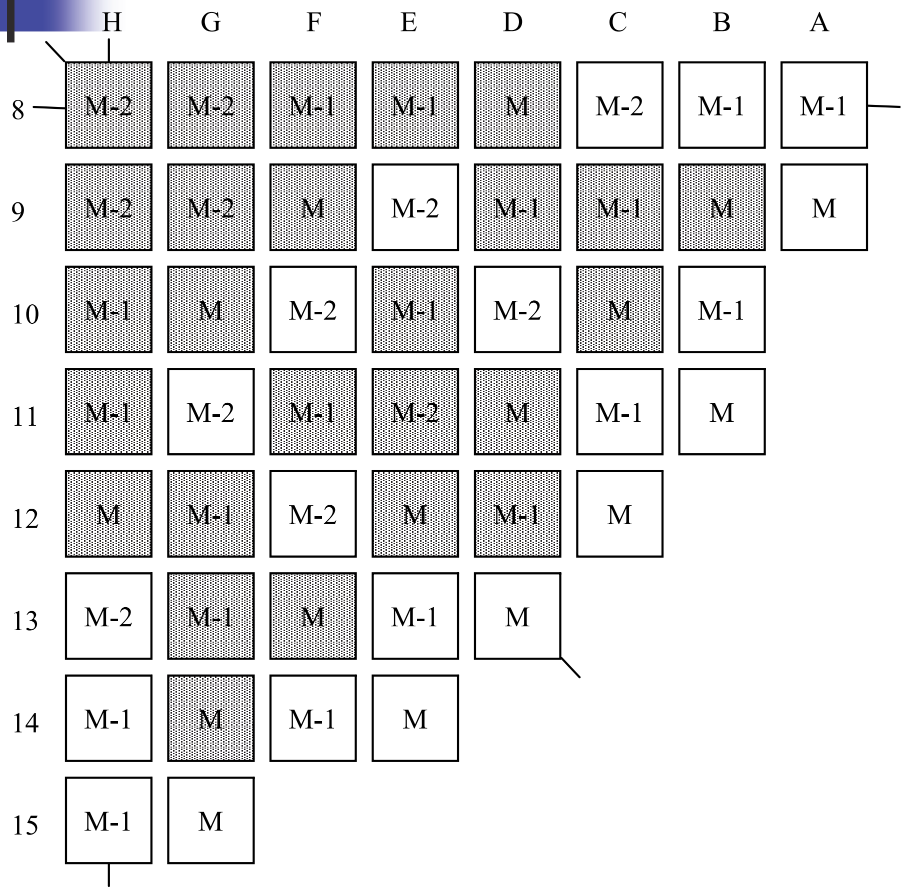
Reload core analyses: Analyses flow for PWR

- Select feed and irradiated fuel assemblies for next cycle
  - The number of feed assemblies in the next cycle is estimated based on the plant-operating plan.
  - Inventory of the burnable poison, which affects the moderator temperature coefficient, is also investigated.
- Perform a rough feasibility study for successive cycles
  - The rough estimation about feed assemblies for successive two or three cycle is performed to confirm feasibility of these cycles.
  - These calculations are performed before the reactor completed the operation of this cycle.

Reload core analyses: Analyses flow for PWR

- Loading pattern design
  - When the inspection of the irradiated fuels is completed after reactor shutdown, the reload core analyses are performed.
  - Detailed loading pattern design is performed to find out a more economical solution while satisfying the safety criteria.
- Prepare documents for utilities or licensing authorities
  - Safety analyses reports are prepared and submitted to a utility. If the utility approve the loading pattern, the safety analyses reports are submitted to the licensing authority via utilities.
  - A lot of documents that are required for startup testing, reactor operation, and so on, are also prepared.

Reload core analyses: Example of loading pattern of PWR



M-2 (M-2) A Region (4.1wt%(Gd) 13)*

M-2 (M-2) B Region (4.1wt% 24)

M-1 (M-1) A Region (4.1wt%(Gd) 36)*

M-1 (M-1) B Region (4.1wt% 24)

M (M) A Region (4.1wt%(Gd) 36)*

M (M) B Region (4.1wt% 24)

Note) M and M correspond to feed assembly

*) contains 16 fuel rods of 2.6wt%U^{235}-6wt%G_2O_3

13.5 EFPM operation, 60 fresh feed assemblies

Reload core analyses: example of calculation results of PWR

Items			Unit	Limitation	Evaluation results
Shutdown margin			$\% \Delta k / k$	≥1.8	2.13
Maximum linear power			kW/m	≤39.6	≤34.2
Assembly maximum burnup			MWd/t	≤48,000	46,400
F_{XY}^{N} (Radial peaking factor)			–	≤1.48	1.43
Moderator temperature coefficient			10^{-5} $(\Delta k / k)/^{\circ}C$	$-78 \le \frac{\partial \rho}{\partial Tm} \le +8$	-5.9～-0.9
Doppler coefficient			10^{-5} $(\Delta k / k)/^{\circ}C$	$-5.2 \le \frac{\partial \rho}{\partial Tf} \le -1.8$	-3.2～-2.3
RCC Drop	Dropped rod worth		$\% \Delta k / k$	≤0.25	0.15
	$F_{\Delta H}^{N}$		–	≤1.84	1.61
FQ at RCC Ejection	BOC	HZP	–	≤14	7.02
		HFP	–	≤7.0	4.11
	EOC	HZP	–	≤26	22.55
		HFP	–	≤5.6	4.94
Ejected Rod worth	BOC	HZP	$\% \Delta k / k$	≤0.94	0.43
		HFP	$\% \Delta k / k$	≤0.19	0.13
	EOC	HZP	$\% \Delta k / k$	≤1.0	0.85
		HFP	$\% \Delta k / k$	≤0.19	0.15
Maximum reactivity insertion			10^{-5} $(\Delta k / k)/^{\circ}C$	≤86	64

(Note)

HZP : Hot zero power, HFP : Hot full power

BOC: Beginning of cycle, EOC: End of cycle

Reload core analyses: Safety parameters for PWR

- Shutdown margin : Negative reactivity of a core at hot zero power and one-rod stuck condition. Prevents re-criticality due to moderator temperature drop during main steam line break (MSLB).
- Maximum linear power : Prevents fuel melt at the loss-of-coolant accident (LOCA). (Decay heat is proportional to linear power density and excess decay heat may cause fuel melt before re-flooding)
- Assembly maximum burnup : Guarantee fuel intactness. Violation of the assembly maximum burnup limitation may results in excessive cladding corrosion or inner pressure.

Reload core analyses: Safety parameters for PWR

- Radial peaking factor : Maximum relative pin-power (axially integrated). Prevents DNB
- Moderator temperature coefficient : Guarantee negative feedback effect; required for inherent safety during reactor operation
- RCC drop (worth and F-dh) : Prevents DNB at a RCC dropped configuration caused by a malfunction of RCC drive mechanism.

Reload core analyses: Safety parameters for PWR

- RCC ejection (worth and F-dh) : Prevents fuel melt and guarantee intactness of reactor pressure boundary during rapid reactivity insertion accident (RIA) caused by RCC ejection.
- Maximum reactivity insertion rate : Guarantee intactness of fuel and reactor pressure boundary at slow reactivity insertion accident caused by uncontrolled RCC withdrawal.

Core follow and operation: Reactor physics testing for PWR

- Verify nuclear design
  - Approach to critical by boron dilution and control rod withdrawal
  - Measurements of critical boron concentration
  - Measurements of control rod worths by boron dilution method or DRWM
  - Measurements of moderator temperature coefficients by perturbing temperature of primary coolant
  - Measurement of radial power distribution by movable detector

- Took 1~2 days

Core follow and operation: Core tracking

- Confirm validity of nuclear design
- Confirm sound burnup of fuel assemblies
  - In-core power measurement using movable detector (MD) (once per month)
  - On-line core monitoring

Basic theory of in-core fuel management

In-core fuel management tasks in PWR

Basic theory of in-core fuel management

- Linear reactivity model
- Cycle length, discharge burnup and batch size
- Analysis of power sharing

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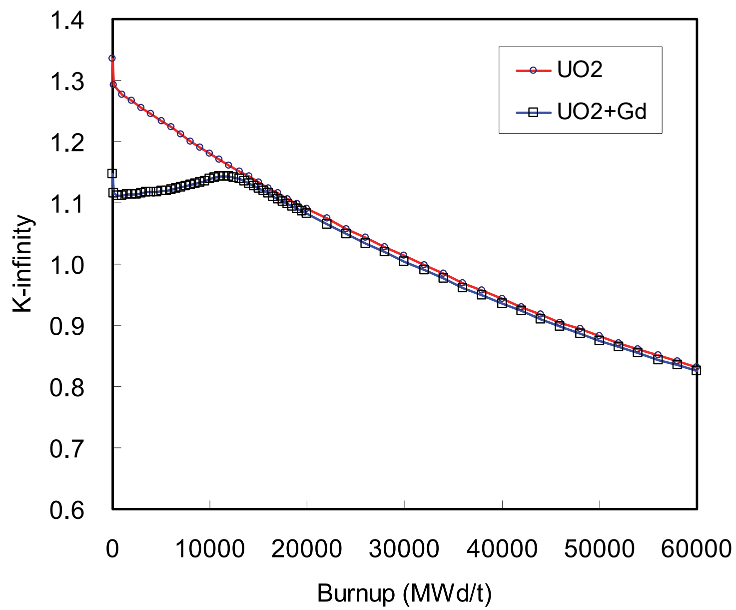
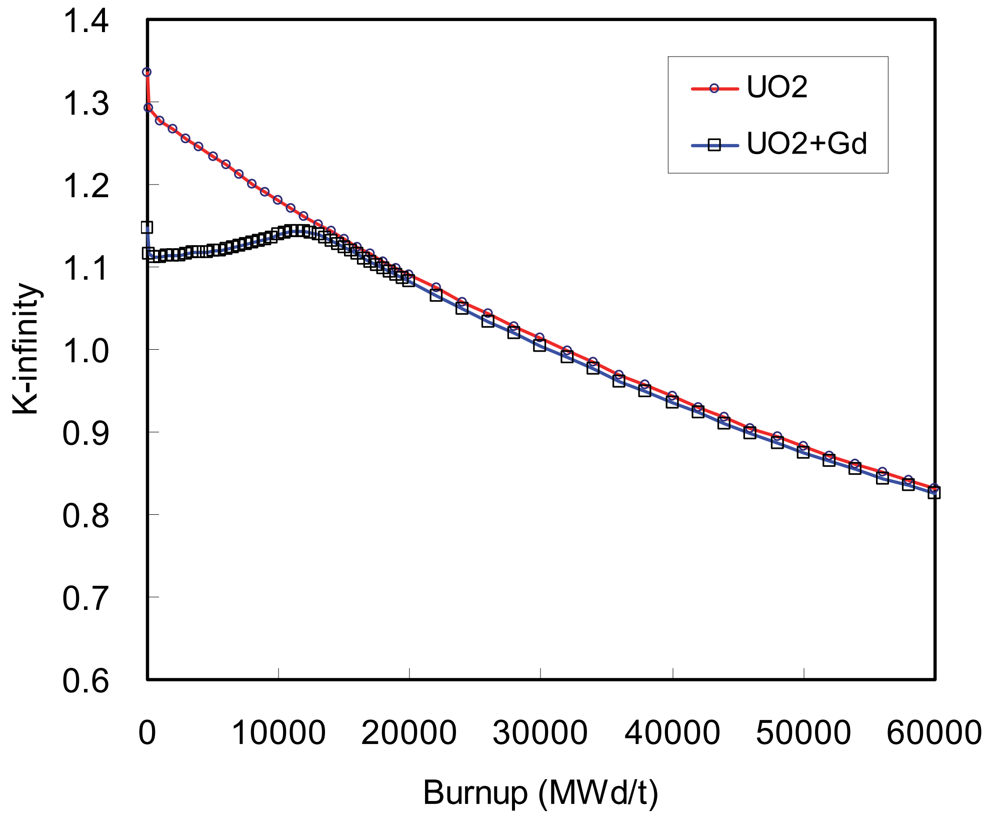


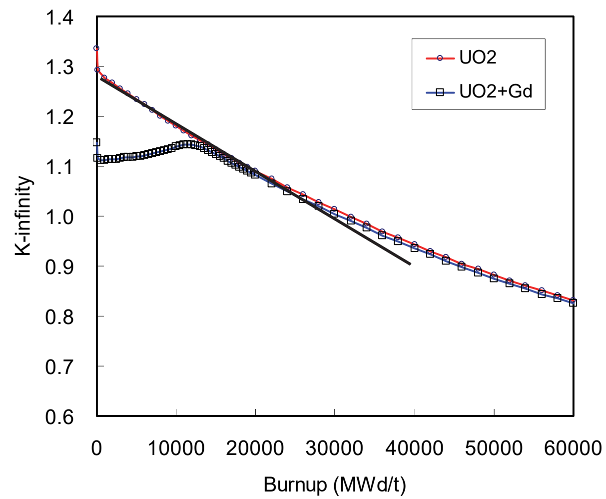
Reactivity variation of fuel assembly during burnup

- PWR 4.1wt% UO2
- 4.1wt%UO2+6wt%Gd2O3



Linear reactivity model (M. J. Driscoll et al., 1990)

- Variation of k-infinity of a fuel assembly without burnable poison is well approximated by a linear function
- Since burnable poison burns-out at EOC, variation of k-infinity of poisoned fuel assembly can be also approximated by a linear function at EOC
- The linear reactivity model can be used for estimation of cycle length



Linear reactivity model (LRM)

- K-infinity or reactivity of a fuel assembly is approximated by a linear function

$$k(B) = -aB + k_0,$$

where,

$k(B)$: k - infinity at burnup B,

k_0 : k - infinity at BOL (B = 0).

$$\rho(B) = -aB + \rho_0,$$

where,

$\rho(B)$: reactivity at burnup B,

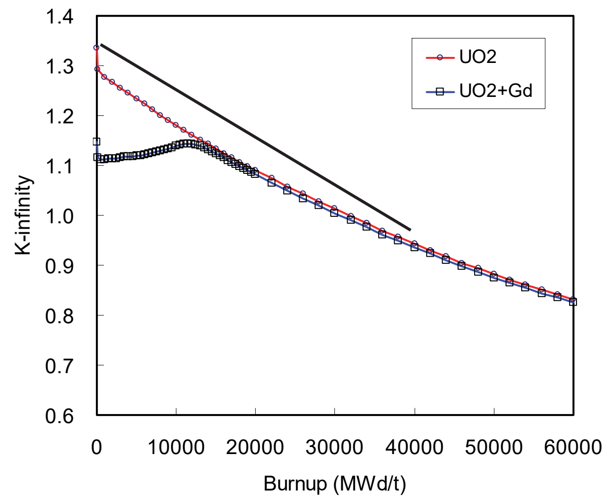
ρ_0 : reactivity at BOL (B = 0).

For example, for 4.1wt% fuel assembly,

$$k(B) \approx -0.01B + 1.3$$

$$\rho(B) \approx -0.01B + 0.3$$

B : fuel burnup in GWd/t



Fuel batch

- Fuel batch is a group of fuel assemblies loaded at the same cycle
- Number of batch (N)

$$N = \frac{\text{Total number of fuel assemblies in a core}}{\text{Number of fresh fuel assemblies}}$$

- For example…
  - One-third of fuel assemblies are replaced with fresh fuel assemblies in a three-batch core

Analysis of a one-batch core by LRM

- Burnup of each batch

	Burnup at BOC	Burnup at EOC
Batch 1	0	C

- Cycle length (C)

$$\rho(C) = -aC + \rho_0 = 0$$

$$\therefore C = \frac{\rho_0}{a}$$

For example,

$$\rho(C) = -0.01C + 0.3 = 0$$

$$\therefore C = \frac{0.3}{0.01} = 30[\mathrm{GWd/t}]$$

- Discharge burnup (DB)

$$DB = C = \frac{\rho_0}{a}$$

For example,

$$DB = 30[\mathrm{GWd/t}]$$

Analysis of a two-batch core by LRM

- Burnup of each batch

	Burnup at BOC	Burnup at EOC
Batch 1	0	C
Batch 2	C	2C

- Core reactivity at cycle length C, assuming uniform power sharing (1.0)

$$\rho(C) = \frac{\rho_{Batch1}(C) + \rho_{Batch2}(C)}{2}$$

$$= \frac{\left(-aC + \rho_0\right) + \left(-a(2C) + \rho_0\right)}{2}$$

Analysis of a two-batch core by LRM

- Cycle length (C)

$$\rho(C) = \frac{\left(-aC + \rho_0\right) + \left(-a(2C) + \rho_0\right)}{2} = 0$$

$$\therefore C = \frac{2}{3}\frac{\rho_0}{a}$$

For example,

$$C = \frac{2}{3}\frac{0.3}{0.01} = 20[\mathrm{GWd/t}]$$

- Discharge burnup (DB)

$$DB = 2C = \frac{4}{3}\frac{\rho_0}{a}$$

For example,

$$DB = 40[\mathrm{GWd/t}]$$

Analysis of a three-batch core by LRM

- Burnup of each batch

	Burnup at BOC	Burnup at EOC
Batch 1	0	C
Batch 2	C	2C
Batch 3	2C	3C

- Cycle length (C)

$$\rho(C) = \frac{\left(-aC+\rho_0\right)+\left(-a(2C)+\rho_0\right)+\left(-a(3C)+\rho_0\right)}{3} = 0$$

$$\therefore C = \frac{1}{2}\frac{\rho_0}{a}$$

For example,

$$C = \frac{1}{2}\frac{0.3}{0.01} = 15[\mathrm{GWd/t}]$$

- Discharge burnup (DB)

$$DB = 3C = \frac{3}{2}\frac{\rho_0}{a}$$

For example,

$$DB = 45[\mathrm{GWd/t}]$$

Analysis of a N-batch core by LRM

- Burnup of each batch

	Burnup at BOC	Burnup at EOC
Batch 1	0	C
…		
Batch N	(N-1)C	NC

- Cycle length (C)

$$\rho(C) = \sum_{i=1}^{N} \frac{\left(-aiC + \rho_0\right)}{N} = 0$$

$$\therefore C = \frac{2}{N+1}\frac{\rho_0}{a}$$

For example,

$$C = \frac{60}{N+1}[\mathrm{GWd/t}]$$

- Discharge burnup (DB)

$$DB = NC = \frac{2N}{(N+1)}\frac{\rho_0}{a}$$

For example,

$$DB = \frac{60N}{N+1}[\mathrm{GWd/t}]$$

Analysis of a N-batch core by LRM N→∞

- Cycle length (C)

$$\lim_{N\to\infty} C = \lim_{N\to\infty} \frac{2}{N+1}\frac{\rho_0}{a} = 0$$

- Discharge burnup (DB)

$$\lim_{N\to\infty} DB = \lim_{N\to\infty} NC = \lim_{N\to\infty} \frac{2N}{(N+1)}\frac{\rho_0}{a} = 2\frac{\rho_0}{a}$$

For example,

$DB = 60[\mathrm{GWd/t}]$

Relationship among cycle length, discharge burnup, and number of batch

- Cycle length(C) vs. number of batch(N)

$$C = \frac{2}{N+1}\frac{\rho_0}{a} \qquad 1 \le \mathrm{N} < \infty \Leftrightarrow \left(0 \le C \le \frac{\rho_0}{a}\right)$$

- Discharge burnup(DB) vs. number of batch(N)

$$DB = \frac{2N}{N+1}\frac{\rho_0}{a} \qquad 1 \le \mathrm{N} < \infty \Leftrightarrow \left(\frac{\rho_0}{a} \le DB \le 2\frac{\rho_0}{a}\right)$$

- Discharge burnup(DB) vs. cycle length(C)

$$DB = 2\frac{\rho_0}{a} - C \Leftrightarrow DB + C = 2\frac{\rho_0}{a}$$

Relationship among cycle length, discharge burnup, and batch size

- Important observations
  - Cycle length is inversely proportional to the number of batches
  - Discharge burnup improves as the number of batches increases
  - Discharge burnup + cycle length = constant
- Longer cycle operation inevitably decreases discharge burnup, i.e., push up fuel cycle cost under fixed feed enrichment

Consideration on power sharing

- The power sharing of each fuel batch is assumed to be uniform (1.0) so far.
- In reality, non-uniform power sharing among fuel batches should be considered.
- In the following discussion, the power sharing of each fuel batch is considered as independent variable
  - We can freely “adjust” the power sharing of each batch.

Optimum power sharing for equilibrium cycle (three-batch case)

- Burnup of each batch

	Power sharing	Burnup at BOC	Burnup at EOC
Batch 1	P_1	0	P_1C_e
Batch 2	P_2	P_1C_e	$(P_1+P_2)C_e$
Batch 3	P_3	$(P_1+P_2)C_e$	$(P_1+P_2+P_3)C_e$

- Cycle length

$$\rho(C_e) = \frac{P_1\rho(P_1C_e) + P_2\rho((P_1+P_2)C_e) + P_3\rho((P_1+P_2+P_3)C_s)}{3}$$

$$= \frac{P_1(-aP_1C_e+\rho_0) + P_2(-a(P_1+P_2)C_e+\rho_0) + P_3(-a(P_1+P_2+P_3)C_s+\rho_0)}{3}$$

$$= 0$$

$$\therefore C_e = \frac{3\rho_0}{a(P_1^2+P_2^2+P_3^2+P_1P_2+P_2P_3+P_1P_3)} \qquad (P_1+P_2+P_3=3)$$

Optimum power sharing for equilibrium cycle (three-batch case)

- Discharge burnup

$$DB = 3C_e = \frac{9\rho_0}{a(P_1^2 + P_2^2 + P_3^2 + P_1P_2 + P_2P_3 + P_1P_3)}$$

- Cycle length and discharge burnup are function of power sharing
- Cycle length and discharge burnup can be maximized by choosing appropriate power sharing

Optimum power sharing for equilibrium cycle (three-batch case)

- Maximization of cycle length

$$C_e = \frac{3\rho_0}{a(P_1^2 + P_2^2 + P_3^2 + P_1P_2 + P_2P_3 + P_1P_3)} \qquad \text{subject to } P_1 + P_2 + P_3 = 3$$

$$\Rightarrow P_1 = P_2 = P_3 = 1$$

$$C_e = \frac{\rho_0}{2a}$$

- Maximization of discharge burnup

$$DB = 3C_e = \frac{9\rho_0}{a(P_1^2 + P_2^2 + P_3^2 + P_1P_2 + P_2P_3 + P_1P_3)}$$

$$\Rightarrow P_1 = P_2 = P_3 = 1$$

$$DB = \frac{3\rho_0}{2a}$$

Optimum power sharing for equilibrium cycle (three-batch case)

- Summary

	Power sharing	Burnup at BOC	Burnup at EOC
Batch 1	1	0	$\rho_0/2a$
Batch 2	1	$\rho_0/2a$	ρ_0/a
Batch 3	1	ρ_0/a	$3\rho_0/2a$

- Example (ρ_0=0.3,a=0.01)

	Power sharing	Burnup at BOC	Burnup at EOC
Batch 1	1	0 GWd/t	15 GWd/t
Batch 2	1	15 GWd/t	30 GWd/t
Batch 3	1	30 GWd/t	45 GWd/t

$$C_e = 15[\mathrm{GWd/t}]$$

$$\mathrm{DB} = 45[\mathrm{GWd/t}]$$

Optimum power sharing for single cycle (three-batch case)

- Burnup of each batch

	Power sharing	Burnup at BOC	Burnup at EOC
Batch 1	P_1	0	P_1C_s
Batch 2	P_2	B_1	$B_1+P_2C_s$
Batch 3	P_3	B_2	$B_2+P_3C_s$

- Cycle length

$$\rho(C_s) = \frac{P_1\rho(P_1C_s) + P_2\rho(B_1 + P_2C_s) + P_3\rho(B_2 + P_3C_s)}{3}$$

$$= \frac{P_1\left(-aP_1C_s + \rho_0\right) + P_2\left(-a(B_1 + P_2C_s) + \rho_0\right) + P_3\left(-a(B_2 + P_3C_s) + \rho_0\right)}{3}$$

$$= 0$$

$$\therefore C_s = \frac{-a(B_1P_2 + B_2P_3) + 3\rho_0}{a(P_1^2 + P_2^2 + P_3^2)} \qquad (P_1 + P_2 + P_3 = 3)$$

Optimum power sharing for single cycle (three-batch case)

- Discharge burnup

$$DB = B_2 + P_3 C_s = B_2 + P_3 \frac{-a(B_1 P_2 + B_2 P_3) + 3\rho_0}{a(P_1^2 + P_2^2 + P_3^2)}$$

- Cycle length and discharge burnup are function of power sharing
- Cycle length and discharge burnup can be maximized by choosing appropriate power sharing

Optimum power sharing for single cycle (three-batch case)

- Maximize cycle length
  - Relation between cycle length and power sharing

$$C_s = \frac{-a(B_1P_2 + B_2P_3) + 3\rho_0}{a(P_1^2 + P_2^2 + P_3^2)}$$

$$\text{subject to } P_1 + P_2 + P_3 = 3,$$

$$B_1 = 15,\ B_2 = 30,\ \text{(taken from equilibrium core)},$$

$$\rho_0 = 0.3,\ a = 0.01$$

  - Result

$$C_s = 17.183[\text{GWd/t}]$$

$$\text{DB} = 39.683[\text{GWd/t}]$$

$$P_1 = 1.436,\ P_2 = 1.000,\ P_3 = 0.564$$

Optimum power sharing for single cycle (three-batch case)

- Maximize discharge burnup
  - Relation between discharge burnup and power sharing

$$DB = B_2 + P_3 C_s = B_2 + P_3 \frac{-a(B_1 P_2 + B_2 P_3) + 3\rho_0}{a(P_1^2 + P_2^2 + P_3^2)}$$

$$\text{subject to } P_1 + P_2 + P_3 = 3,\ B_1 = 15,\ B_2 = 30,\ \rho_0 = 0.3,\ a = 0.01$$

  - Result

$$C_s = 13.090[\text{GWd/t}]$$

$$\text{DB} = 46.771[\text{GWd/t}]$$

$$P_1 = 1.146, P_2 = 0.573, P_3 = 1.281$$

Comparison with equilibrium and single cycle optimum power sharing

- Calculation conditions
  - LRM $\rho(C) = -0.01C + 0.3$
  - Three batch
- Results

Case	Maximize	Cycle length (GWd/t)	Discharge burnup (GWd/t)	Power sharing		
				Fresh	1ce burnt	2ce burnt
Single cycle	Cycle length	17.183	39.683	1.436	1.000	0.564
	Discharge burnup	13.090	46.771	1.146	0.573	1.281
Equilibrium cycle	Cycle length	15.000	45.000	1.000	1.000	1.000
	Discharge burnup	15.000	45.000	1.000	1.000	1.000

“Paradox” in single cycle optimization

- Cycle length and discharge burnup have trade-off relationship in single cycle analysis
- Increasing cycle length causes lower discharge burnup
  - Lower discharge burnup push up number of fresh fuels in successive cycles
- Increasing discharge burnup causes shorter cycle length
  - Shorter cycle length push up number of fresh fuel assembly in the current cycle
- Which strategy is “better” from the viewpoint of overall (multi-cycle) fuel cost?
- Comparison power sharing in (successive) multi-cycle is necessary

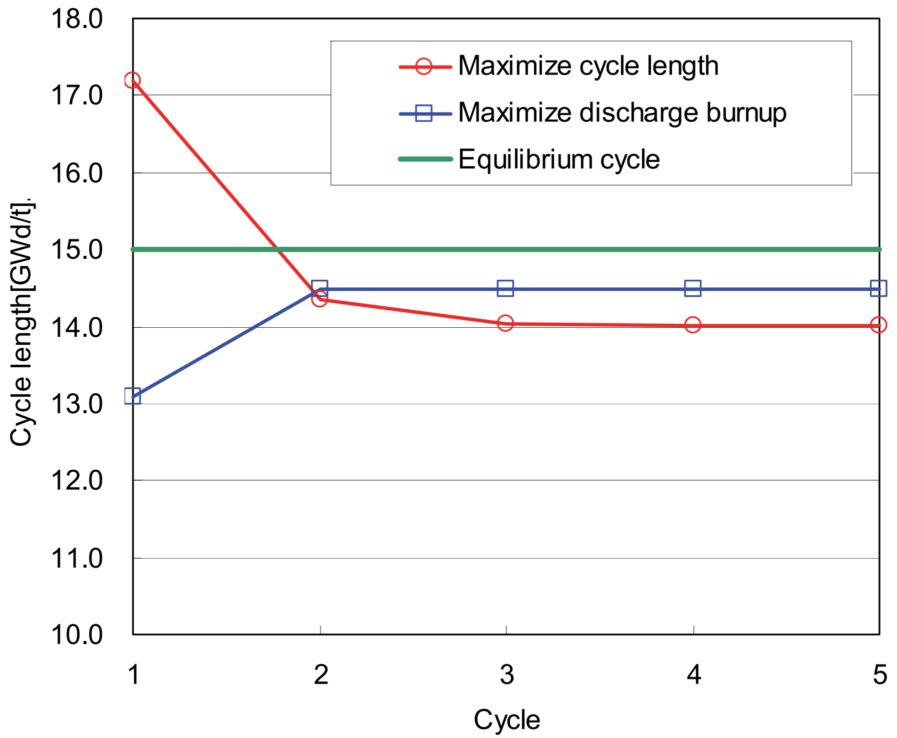
Comparison with successive single-cycle and multi-cycle optimum power sharing

- Compared cases
  - Equilibrium cycle
    - A typical situation of multi-cycle
  - Successive single cycle, maximizing cycle length in each cycle
  - Successive single cycle, maximizing discharge burnup in each cycle

Comparison with successive single-cycle and multi-cycle optimum power sharing

- Cycle length

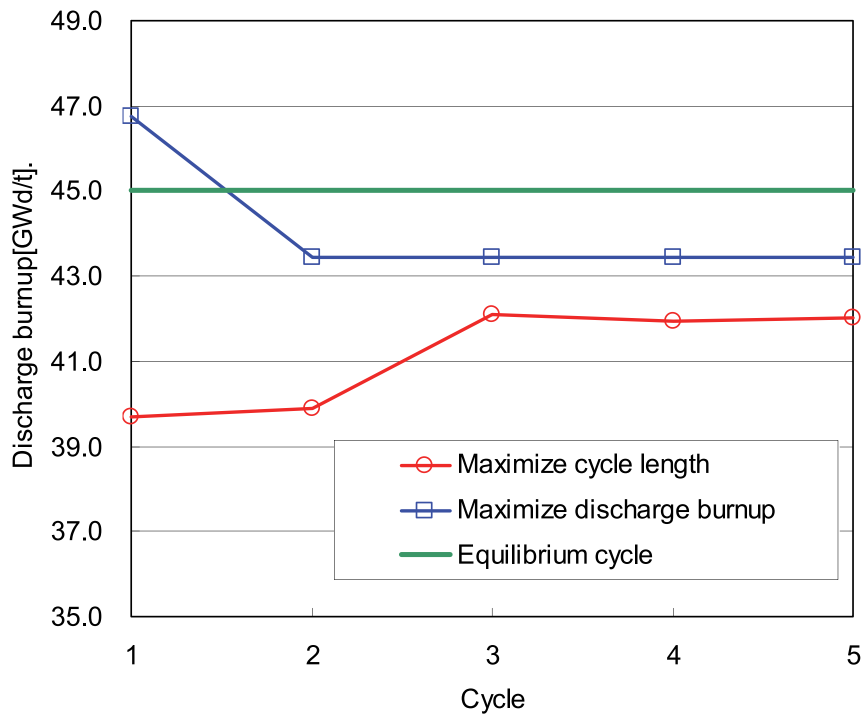
Cycle	Objective fucntion	
	Max. cycle length (GWd/t)	Max. dis. burnup (GWd/t)
Equilibrium	15.000	15.000
1	17.183	13.090
2	14.340	14.478
3	14.023	14.478
4	14.004	14.478
5	14.001	14.478



Comparison with successive single-cycle and multi-cycle optimum power sharing

- Discharge burnup

Cycle	Objective fucntion	
	Max. cycle length (GWd/t)	Max. dis. burnup (GWd/t)
Equilibrium	45.000	45.000
1	39.682	46.771
2	39.909	43.435
3	42.099	43.435
4	41.959	43.435
5	42.003	43.435



Comparison with successive single-cycle and multi-cycle optimum power sharing

- Core average burnup

Cycle	Objective function			
	Max. cycle len(GWd/t)		Max. discharge bu(GWd/t)	
	BOC	EOC	BOC	EOC
Equilibrium	15.000	30.000	15.000	30.000
1	14.999	32.182	15.000	28.090
2	18.955	33.295	12.500	26.978
3	19.992	34.015	12.500	26.978
4	19.982	33.986	12.500	26.978
5	19.999	34.000	12.500	26.978

Comparison with successive single-cycle and multi-cycle optimum power sharing

- Power sharing of each batch

Cycle	Objective function					
	Maximize cycle length			Maximize discharge burnup		
	Fresh	1ce burnt	2ce burnt	Fresh	1ce burnt	2ce burnt
Equilibrium	1.000	1.000	1.000	1.000	1.000	1.000
1	1.436	1.000	0.564	1.146	0.573	1.281
2	1.661	0.800	0.539	1.036	0.518	1.446
3	1.713	0.864	0.423	1.036	0.518	1.446
4	1.713	0.856	0.431	1.036	0.518	1.446
5	1.714	0.857	0.429	1.036	0.518	1.446

Comparison with successive single-cycle and multi-cycle optimum power sharing

- Burnup of each batch at EOC

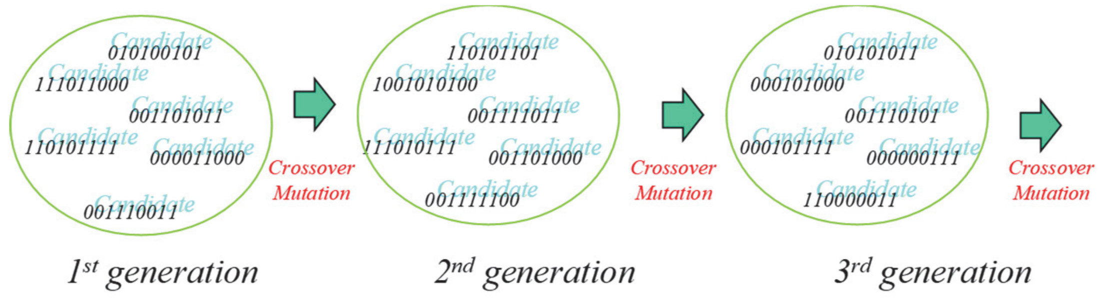
Cycle	Objective function					
	Maximize cycle length(GWd/t)			Maximize discharge burnup(GWd/t)		
	Fresh	1ce burnt	2ce burnt	Fresh	1ce burnt	2ce burnt
Equilibrium	15.000	30.000	45.000	15.000	30.000	45.000
1	24.683	32.183	39.682	15.000	22.500	46.771
2	23.818	36.159	39.909	15.000	22.500	43.435
3	24.019	35.928	42.099	15.000	22.500	43.435
4	23.995	36.005	41.959	15.000	22.500	43.435
5	24.001	35.998	42.003	15.000	22.500	43.435

Multi-cycle coupling effect

- In the ideal (i.e., “true” multi-cycle) situation, maximizations of cycle length and discharge burnup give identical result
- In reality, “true” multi-cycle optimization is difficult and we should use single cycle optimization
- In the single cycle optimization, maximizations of the cycle length and discharge burnup give different results

Multi-cycle coupling effect

- According to the optimum power sharing analysis, maximization of the discharge burnup gives "better" solution than maximization of cycle length.
- A procedure (or an optimization method) for single cycle to find out the "best" solution from the viewpoint of multi-cycle has not been established – we are still looking for a "silver bullet".



Loading pattern optimization

In-core fuel management tasks in PWR

Basic theory of in-core fuel management

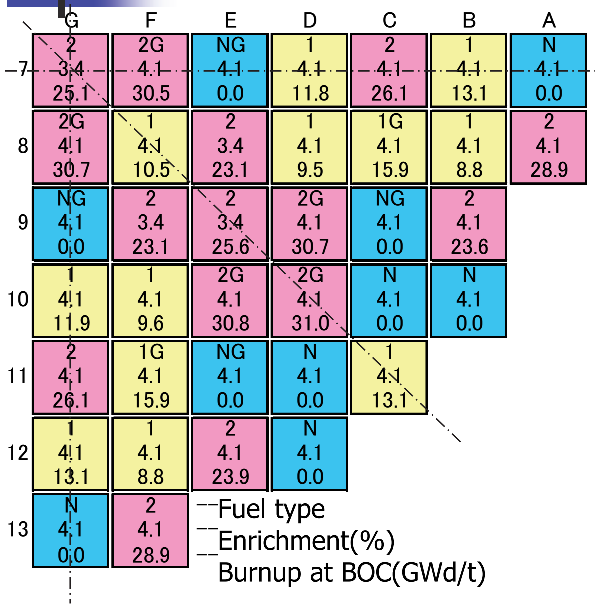
- Linear reactivity model
- Cycle length, discharge burnup and batch size
- Analysis of power sharing

Loading pattern optimizations

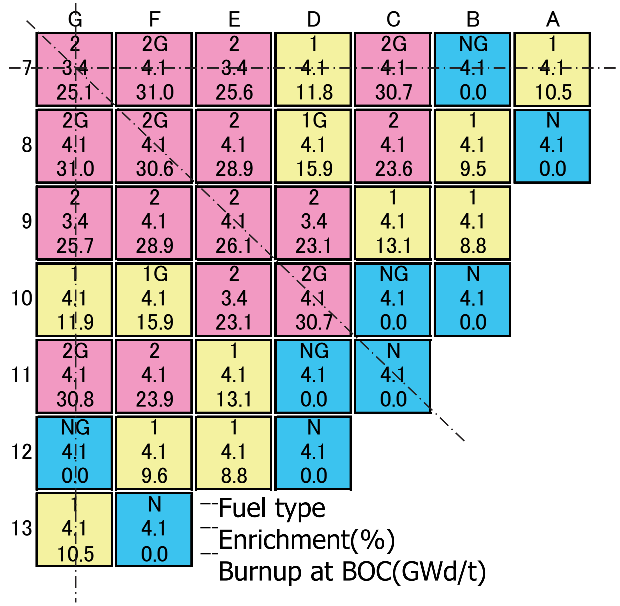
- Overview
- Objectives and constraints
- Simulated annealing/Genetic algorithms/Tabu-search/Heuristics

Summary

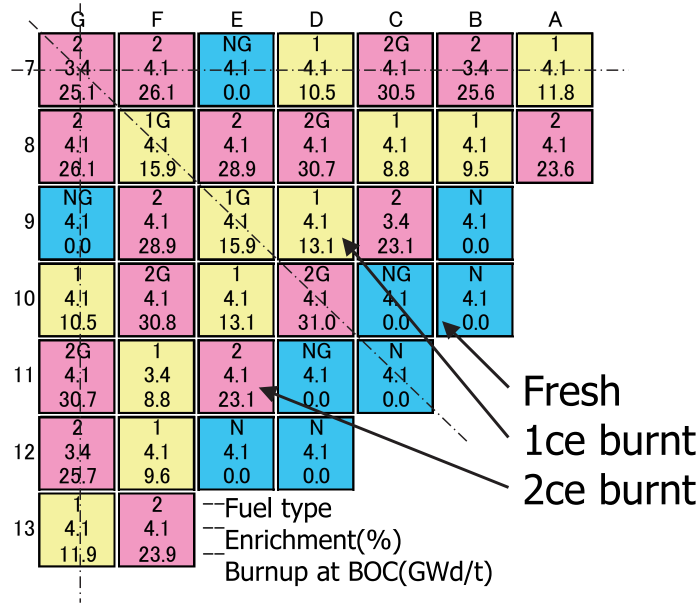
Impact of loading pattern



Core(a)



Core(b)

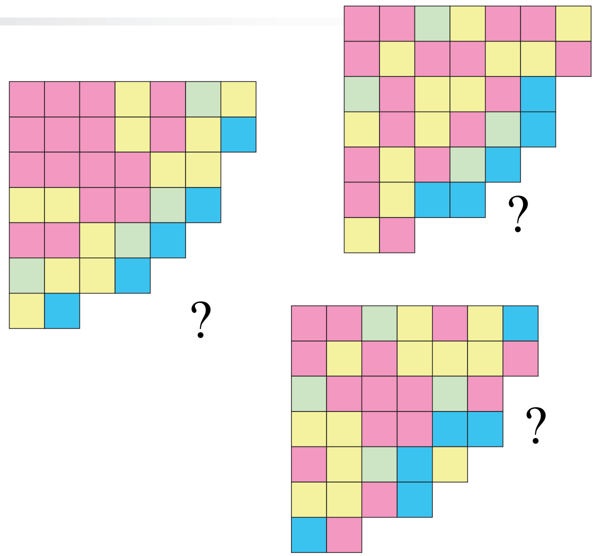


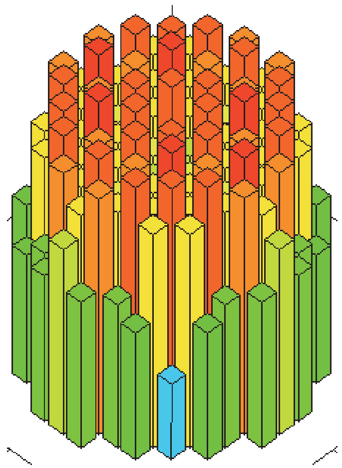
Core(c)

Core characteristics	Limitation	Core (a)	Core (b)	Core (c)
Radial peaking factor	< 1.480	1.572	1.443	1.438
Cycle length (GWd/t)		14.1	12.5	13.4
Discharge burnup (GWd/t)		40.1	38.8	41.2
Fuel cost (billion Yen)		12.4	12.8	12.1

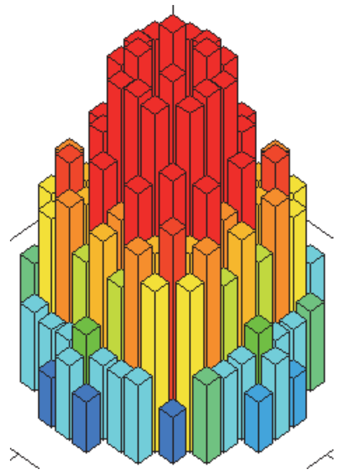
Loading pattern optimization: Overview

- Develop economical loading patterns while satisfying safety and other limitations
- Optimize in-core fuel arrangements
- High industrial needs: a loading pattern must be developed in each cycle
- Optimization is difficult because of its stiff natures
- State-of-art optimizations by engineer are often performed





Proper power distribution



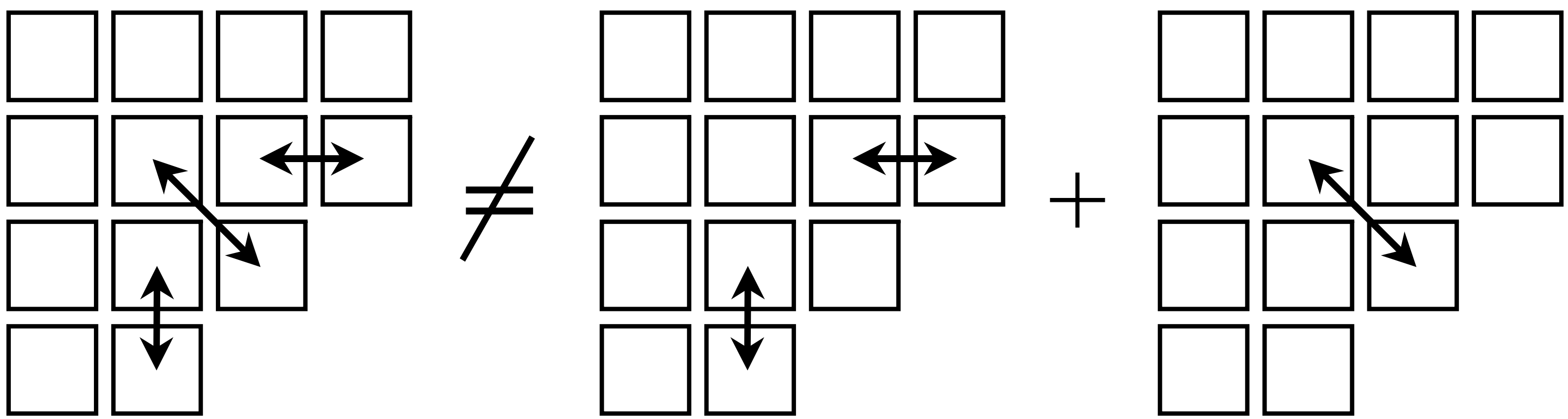
Skewed power distribution

Loading pattern optimization: Features

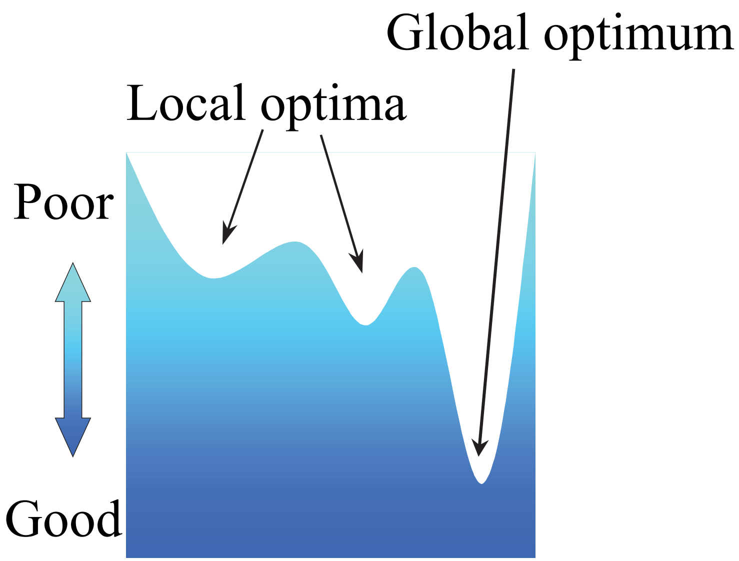
- Combinatorial (or discrete) optimization problem
- Enumeration number is quite large; 10^{20}-10^{30}
- Non-linearity on objective function
- Many local optima
- Multi-objective optimization with many constraints

Non-linearity in core calculation

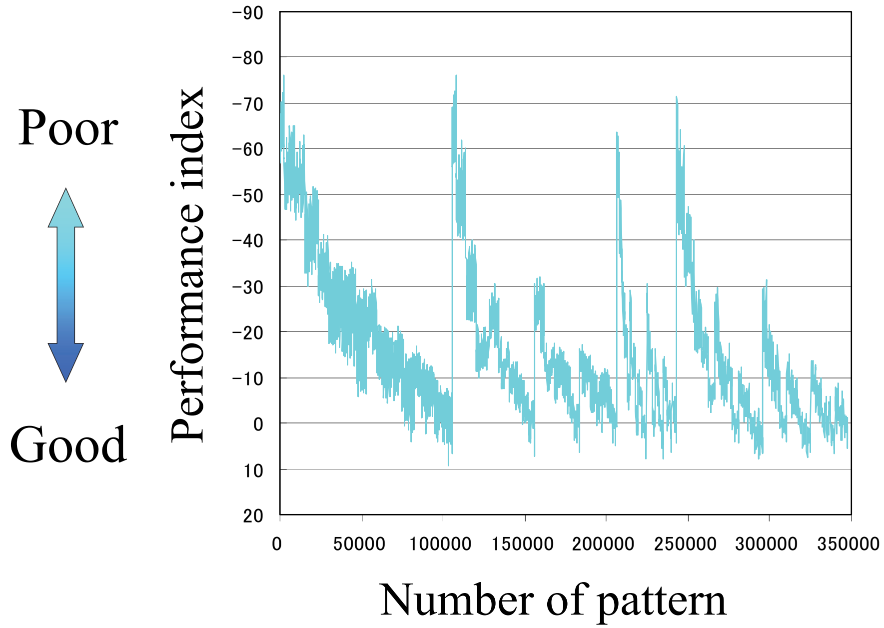
- Linear superposition of assembly power variation gives poor prediction results for PWR



Local optima in loading pattern optimization



Conceptual model



Actual example

Loading pattern optimization: constraints in PWR

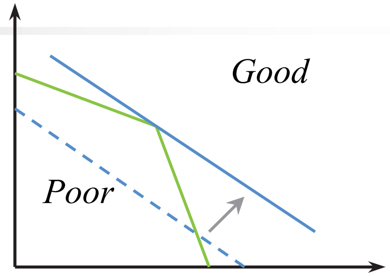
- Radial peaking factor
- Maximum burnup (assembly, pin)
- Moderator temperature coefficient
- Radial power tilt
- Shutdown margin
- Maximum boron concentration
- Clad Induced Power Shift (CIPS)
- Loading positions (Lead Test Assembly…)
- Other safety parameters

Loading pattern optimization: Objectives

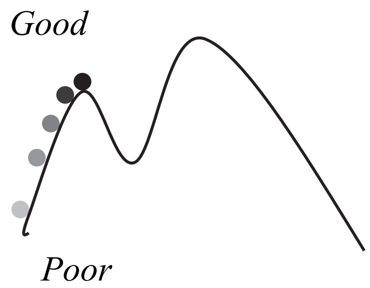
- Maximization of the cycle length under the fixed feed enrichment and the fixed number of fresh fuel.
- Maximization of the discharge burnup under the fixed feed enrichment and the fixed number of fresh fuel.
- Minimization of the number of fresh fuel under the fixed cycle length and the fixed feed enrichment.
- Minimization of the enrichments under the fixed cycle length and the fixed number of fresh fuel.

Loading pattern optimization: Conventional approaches

- Linear programming



- Direct search (hill climb method)
- Enumerated binary exchange



- Artificial intelligence
- Heuristics

```
*
* symmetrical position
*
s01:0:1:(E,1):3.0
s02:0:1:(E,3):2.0
s03:0:1:(E,4):1.0
s04:0:1:(L,4):-1.0
s05:0:1:(E,2):-5.0
s11:1,2,3,4,5,6,7,8,9,10,11:1:(E,4):3.0
s12:1,2,3,4,5,6,7,8,9,10,11:1:(L,3):2.0
```

Loading pattern optimization: Advanced method (stochastic)

- Simulated annealing (SA)
  - SA is an optimization method whose origin is the simulation of the crystal vibration in annealing metal. When a melting metal is annealed slowly, the energy state of a crystal in the solid metal becomes lower than that of quickly annealed metal. SA simulates this process to find out optimum.
- Tabu-search (TS)
  - Similar to SA, but escapes from the searched space by Tabu, which is a short-time memory of searched space.

Loading pattern optimization: Advanced method (stochastic)

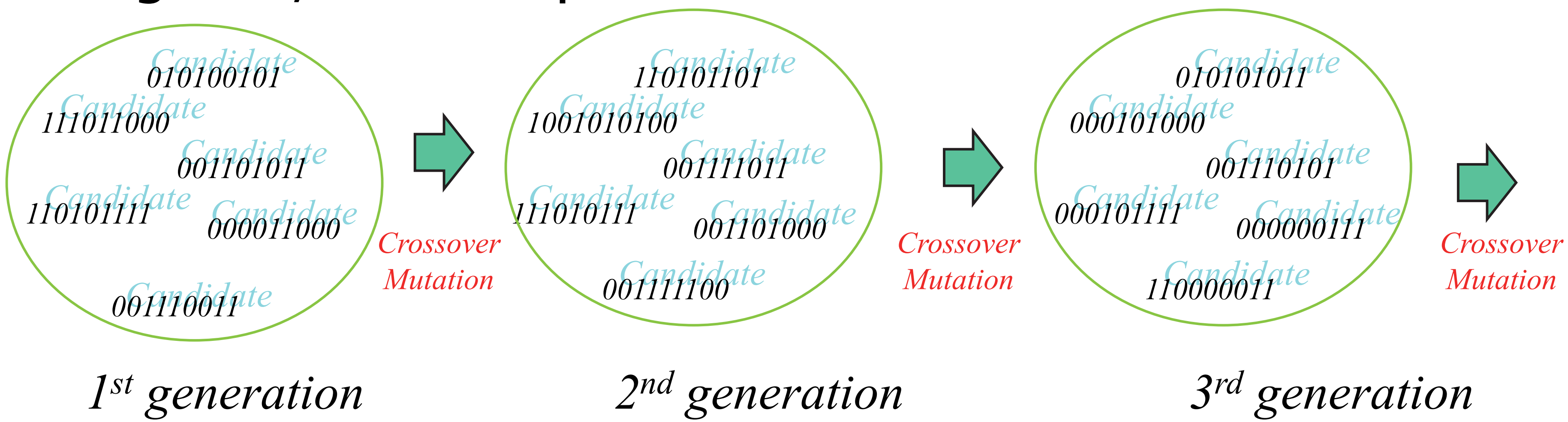
- Genetic algorithms (GA)
  - GA is one of the powerful optimization methods and its concept is based on the genetics and natural selection of life; digital chromosomes that represent solutions evolve based on Darwinian theory.
- Ant-colony search
  - Simulates path-finding strategy of ants to find out optimum solution

Loading pattern optimization: Advanced method (deterministic)

- Batch enumerated search
  - Fuel assemblies are classified into a few to several coarse groups (batches) and all possible combination based on the coarse batches are searched. Then the coarse batch is divided into finer groups. This procedure is repeated to find out a final solution.

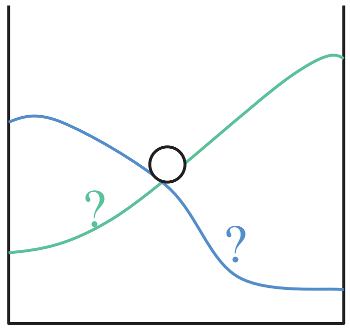
Genetic algorithm

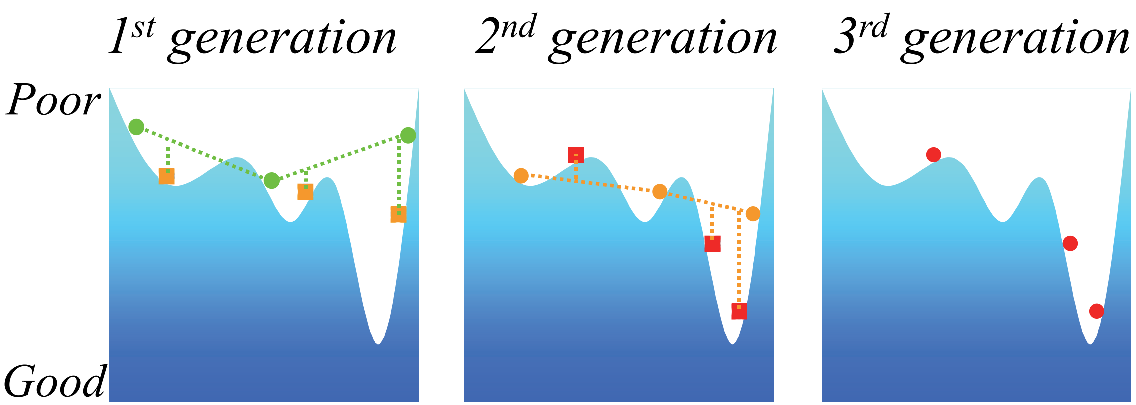
- Genetic algorithm is an optimization method that simulates evolution of life
- Search an optimum solution by digital evolution of genes, which represent candidate solutions



Genetic algorithm

- GA is a “blind watch-maker” – it can be used without sufficient knowledge of the search space, e.g., gradient information
- Only evaluation of the generated gene (i.e. candidate solution) is necessary to perform optimization
- Suitable for global search, but local search capability is not very high

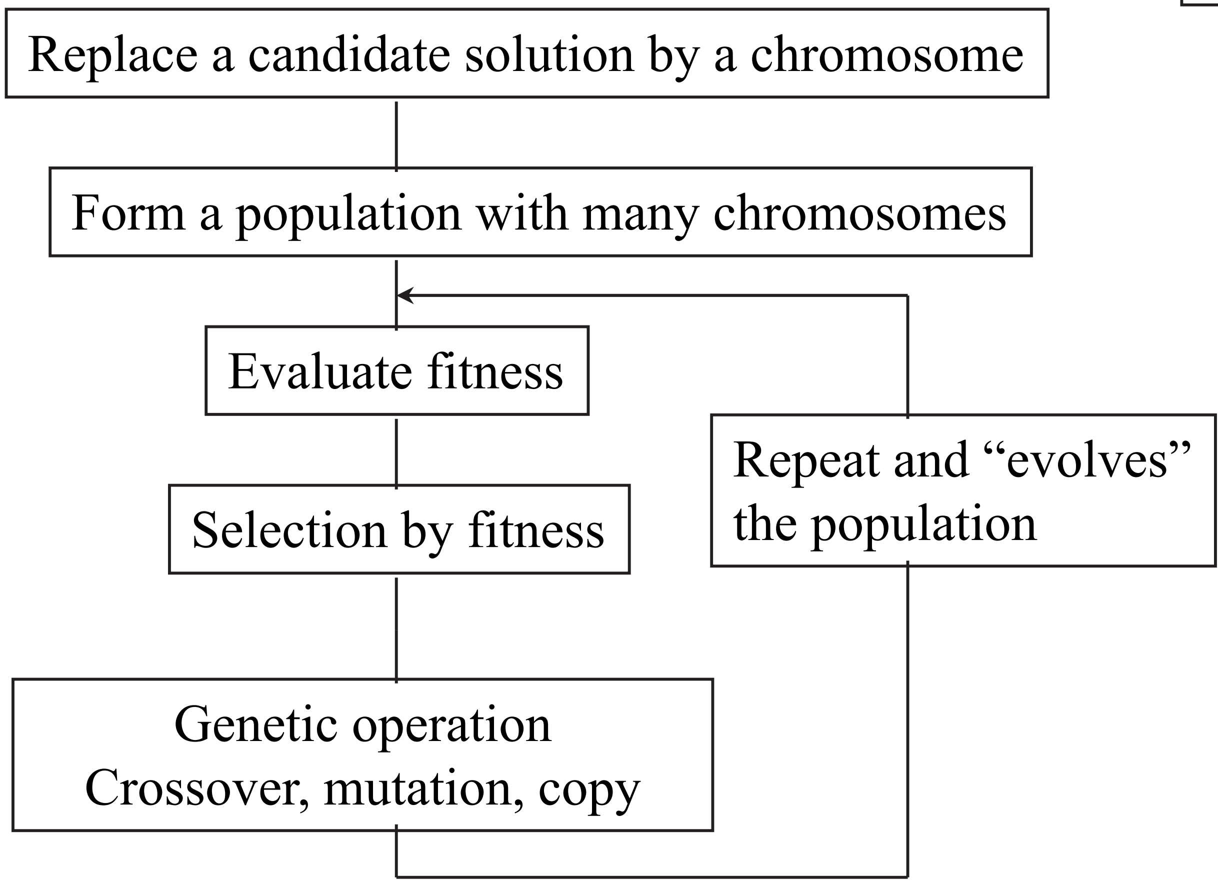


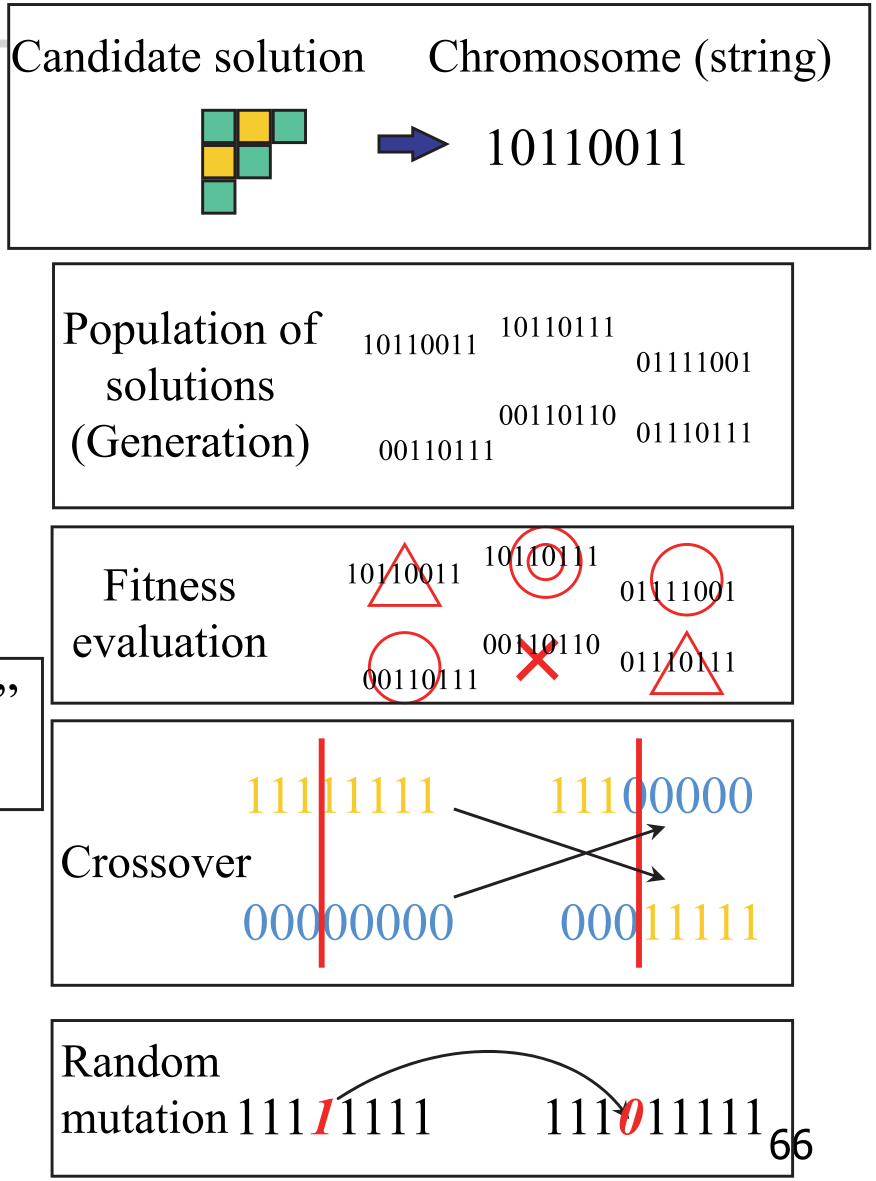


Concept of GA search

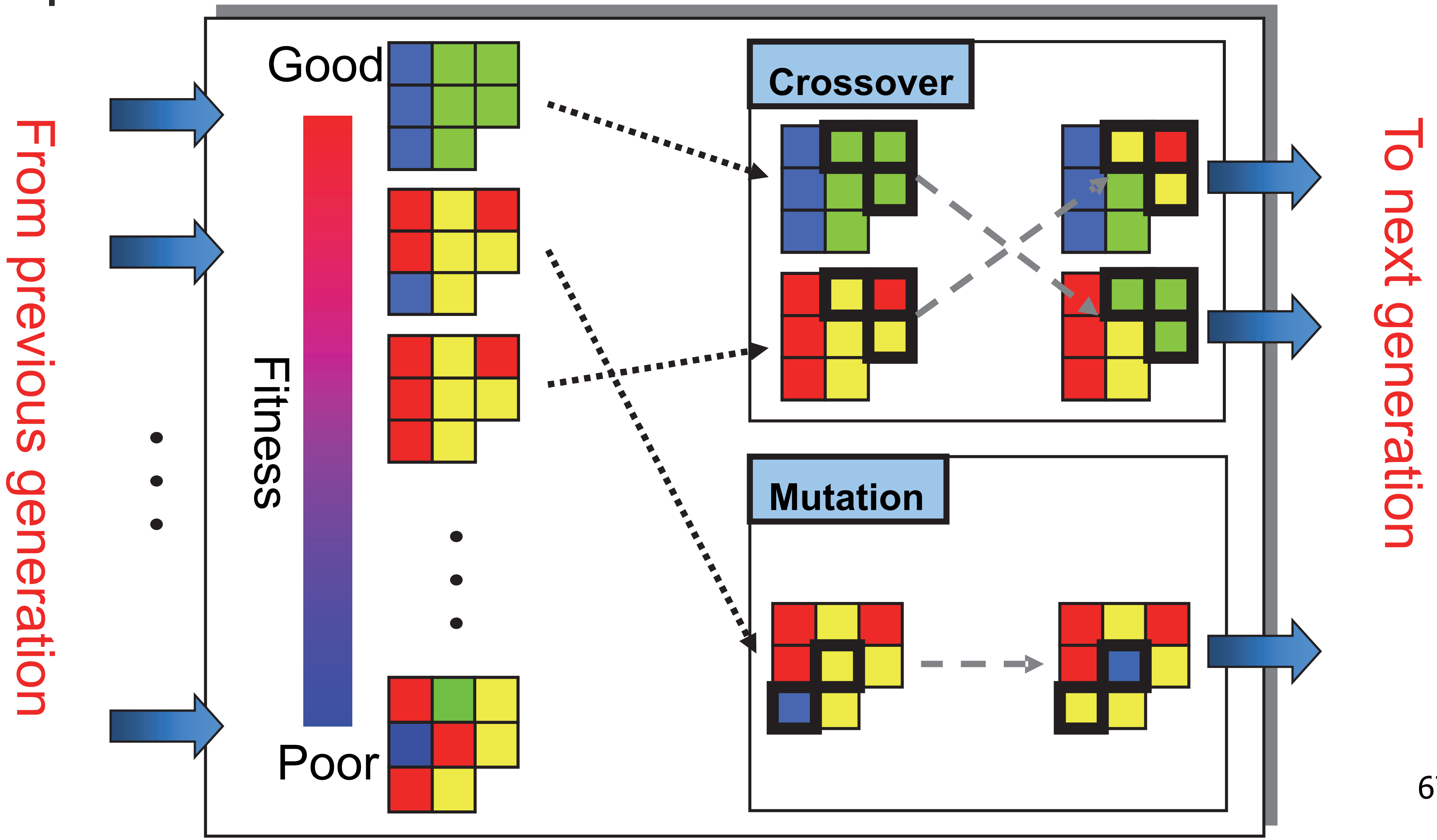
Genetic algorithms

Calculation flow



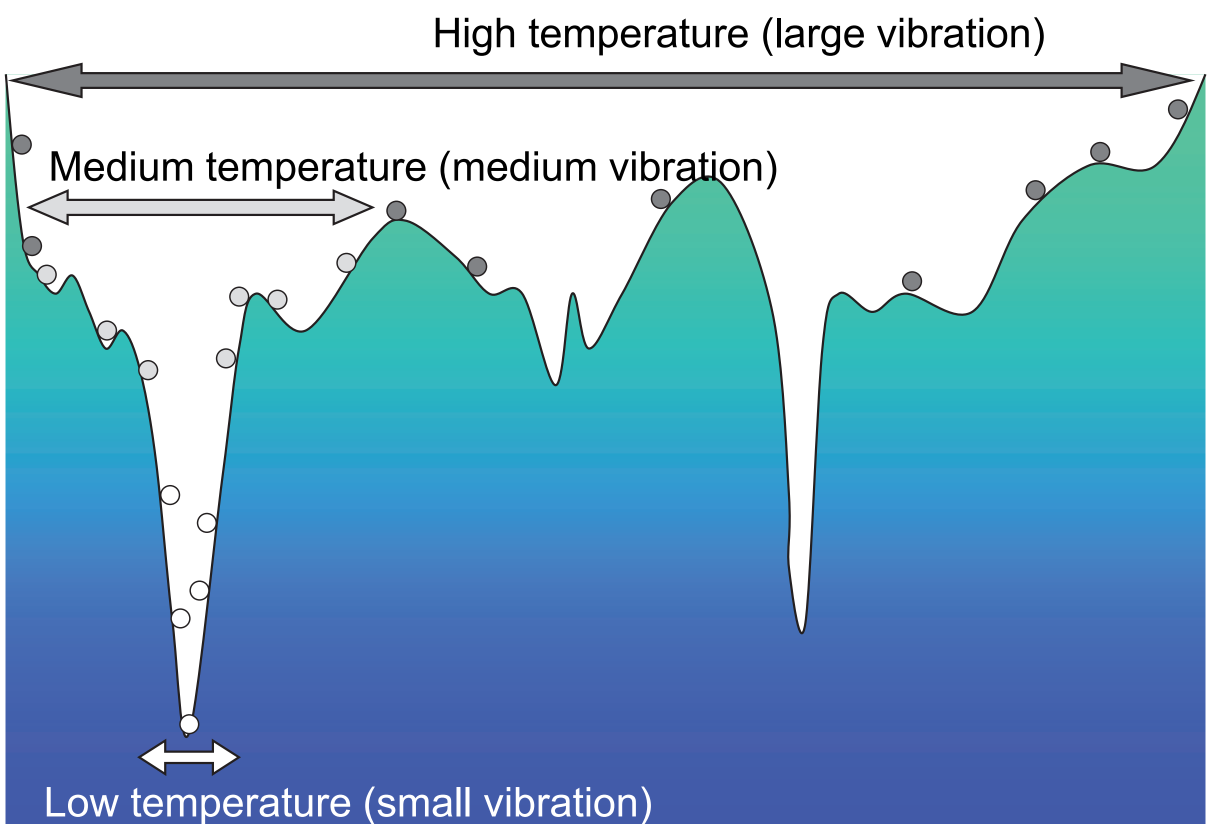


Genetic algorithms



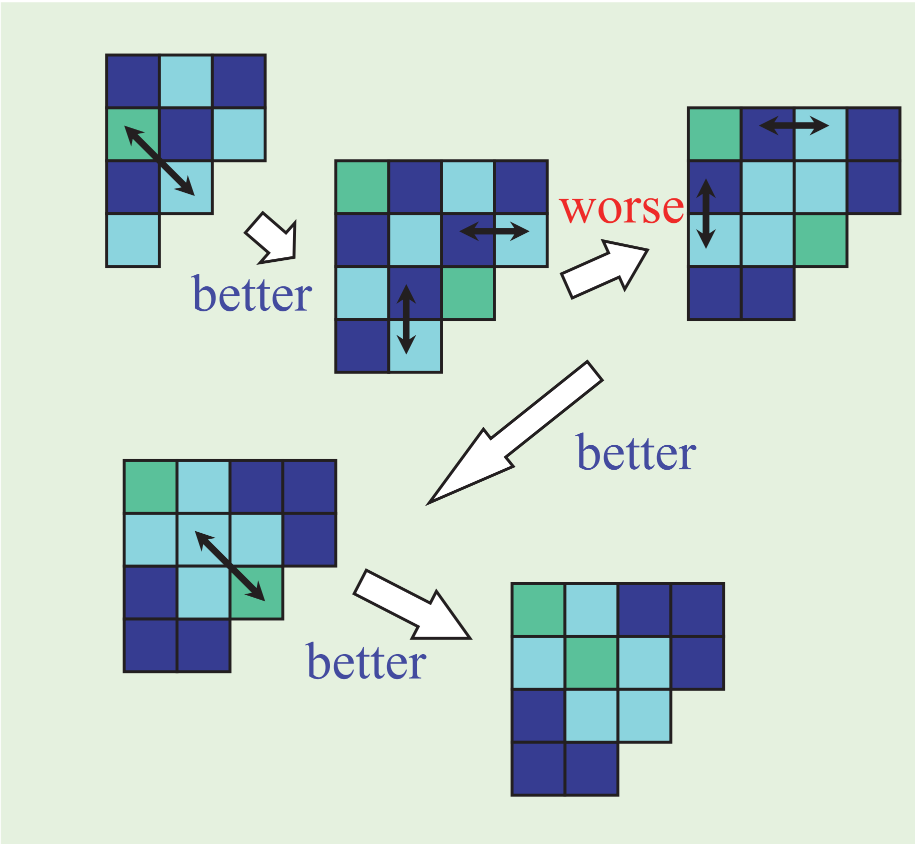
Simulated Annealing, SA

- An optimization method simulating the annealing of metal
  - Accept poor candidate to escape from local optimum solution



Simulated Annealing, SA

- Application to actual loading pattern optimization



$$P = \exp[\frac{\delta F}{T}]$$

P : acceptance prob.

δF : value of obj. func.

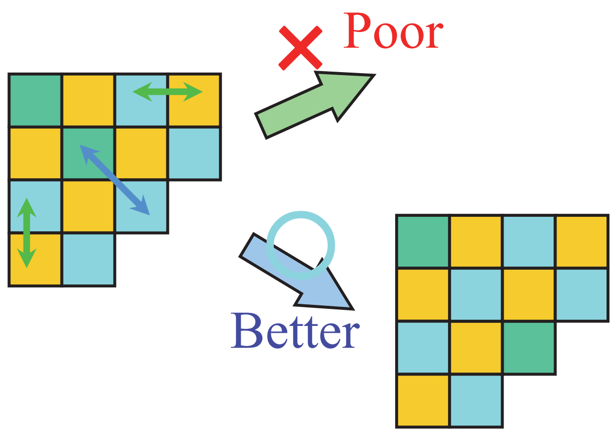
T : system temp.

Tabu-search

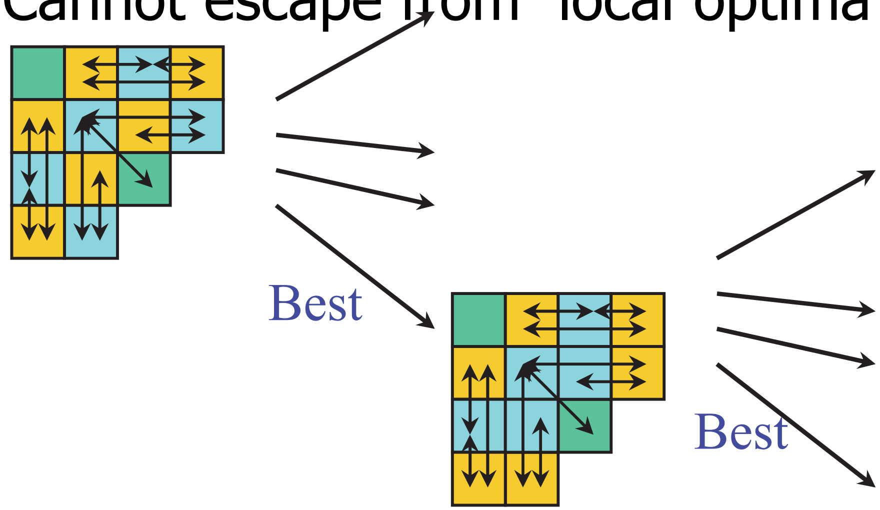
- Similar to SA, but avoid “circulating route” by short-time memory (Taboo)

Direct search, binary exchange

- Direct search method
  - Choose only better solution
  - Cannot escape from local optima

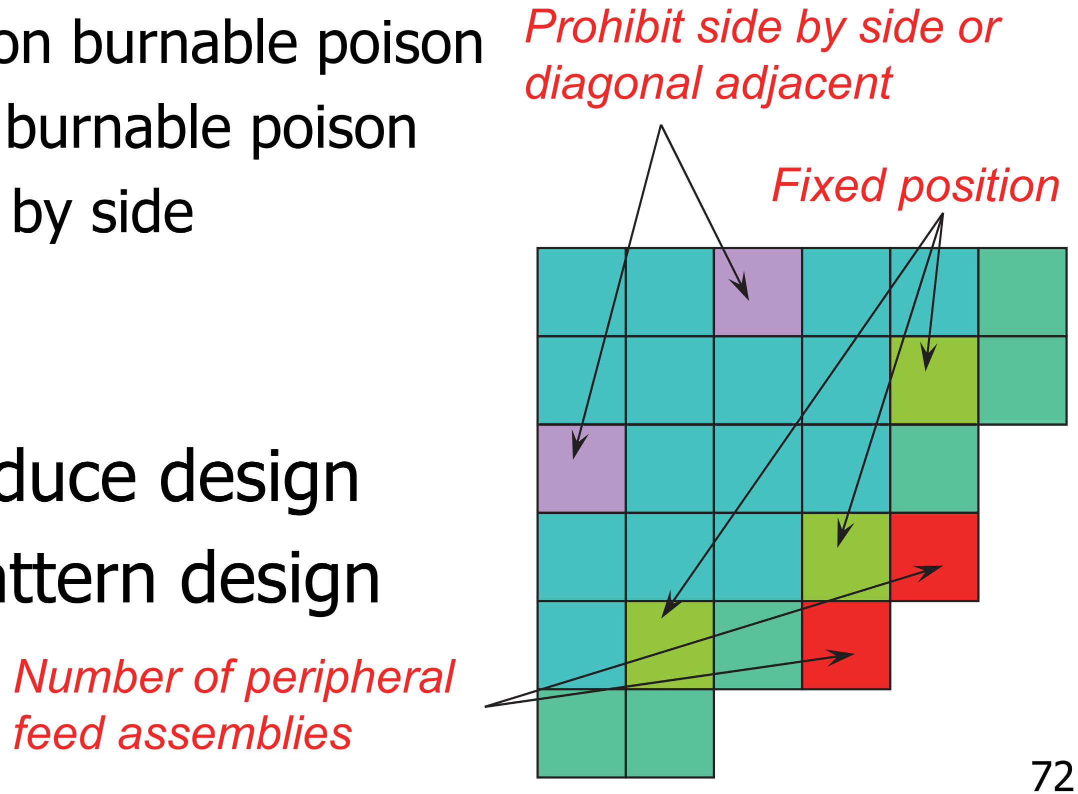


- Binary exchange method
  - Repeat all possible binary swap
  - Cannot escape from local optima



Heuristic rules

- Heuristics is empirical rules for loading pattern design
  - Position restriction on feed assembly
  - Position restriction on burnable poison
  - Fresh fuels without burnable poison are not placed side by side in core inboard
  - …
- Very effective to reduce design space in loading pattern design



Loading pattern optimization: Pros and Cons of advanced method

- Pros
  - Simplicity of algorithms
  - Flexible treatment of constraints and objectives
  - Capability of escaping from local optima

- Cons
  - Longer computation time

Typical evaluations to reach an "optimum" loading pattern

Engineer	***~100 loading patterns***
Direct search	***~1000 loading patterns***
Genetic Algorithms	***~thousands loading patterns***
Simulated Annealing	***~tens of thousands loading patterns***

Loading pattern optimization: Examples

- Target reactor is three loop PWR (157 FAs)
- Feed enrichment: 4.1wt%
- Gadolinia bearing fuel
- Fuel inventory

Serial No.	Burnable Poisons	Burnups (GWd/t)	Number of Fuel Assemblies
1	Gd*)	34.7	1
2	Gd	34.7	4
3	Gd	32.7	4
4	Gd	16.8	4
5		23.0	4
6	Gd	19.5	4
7	Gd	32.2	4
8	Gd	16.8	4
9		28.9	4
10		12.6	4
11		12.6	4
12	Gd	0.0	4
13		23.8	8

Serial No.	Burnable Poisons	Burnups (GWd/t)	Number of Fuel Assemblies
14	Gd	18.9	8
15	Gd	19.0	8
16		27.9	8
17	Gd	18.4	8
18		11.3	8
19		10.2	8
20	Gd	0.0	8
21	Gd	0.0	8
22	Gd	0.0	8
23	Gd	0.0	8
24		0.0	8
25		0.0	8
26		0.0	8

*) Gd shows Gadolinia bearing fuel.

Loading pattern optimization: Examples

- Optimization cases
  - Case 1: minimize the radial peaking factor (Fxyn),
  - Case 2: maximize the cycle length,
  - Case 3: maximize the discharge burnup,
- 3000 loading patterns were evaluated during optimization by Genetic Algorithms

Loading pattern optimization: Examples

- Summary of calculation results

Case	Cycle Length (GWd/t)	Maximum Burnup (GWd/t)	Fxyn	Moderator Temperature Coefficient (pcm/℃)	Discharge Burnup (GWd/t)	Remarks
1	15.304	47.118	1.3565	-3.9	39.661	Minimize Fxyn
2	16.349	46.898	1.4771	-0.6	38.252	Maximize Cycle Length
3	15.282	47.305	1.4777	-0.9	40.495	Maximize Discharge Burnup

Loading pattern optimization: Examples

- Optimized loading patterns for Case 1 and 2

2G 34.7				
2 23.0	2G 34.7			
1G 16.8	NG 0.0	2 28.9		
1G 16.8	2 23.8	1G 18.9	1G 19.5	
2G 32.7	NG 0.0	2 27.9	1G 19.0	NG 0.0
2G 32.2	1G 18.4	1 11.3	NG 0.0	N 0.0
1 12.6	NG 0.0	1 10.2	N 0.0	
1 12.6	N 0.0	--Fuel Type --Fuel assembly burnup(GWd/t)		

N	Fresh fuel without Gd.
1	Once burned fuel without Gd.
2	Twice burned fuel without Gd.
NG	Fresh fuel with Gd.
1G	Once burned fuel with Gd.
2G	Twice burned fuel with Gd.

The calculated loading pattern of Case 1 (Minimize Fxyn) in the benchmark problem

2G 34.7				
2G 32.7	2 23.0			
1G 16.8	NG 0.0	1G 19.5		
NG 0.0	1G 18.9	2 27.9	1G 16.8	
2G 32.2	1G 19.0	1G 18.4	NG 0.0	1 12.6
1 12.6	1 10.2	NG 0.0	1 11.3	N 0.0
2 28.9	NG 0.0	N 0.0	N 0.0	
2G 34.7	2 23.8	--Fuel Type --Fuel assembly burnup(GWd/t)		

N	Fresh fuel without Gd.
1	Once burned fuel without Gd.
2	Twice burned fuel without Gd.
NG	Fresh fuel with Gd.
1G	Once burned fuel with Gd.
2G	Twice burned fuel with Gd.

The calculated loading pattern of Case 2 (Maximize cycle length) in the benchmark problem

Loading pattern optimization: Examples

- Optimized loading patterns for Case 3

2G 34.7				
1G 16.8	2G 32.2			
2 28.9	NG 0.0	2 23.0		
1 12.6	1G 18.9	1G 18.4	NG 0.0	
2G 32.7	NG 0.0	1G 19.0	2 23.8	1G 19.5
1 12.6	2 27.9	1 10.2	NG 0.0	N 0.0
2G 34.7	NG 0.0	1 11.3	N 0.0	
1G 16.8	N 0.0	--Fuel Type --Fuel assembly burnup(GWd/t)		

N	Fresh fuel without Gd.
1	Once burned fuel without Gd.
2	Twice burned fuel without Gd.
NG	Fresh fuel with Gd.
1G	Once burned fuel with Gd.
2G	Twice burned fuel with Gd.

The calculated loading pattern of Case 3 (Maximize discharge burnup) in the benchmark problem

Loading pattern optimization: Comparison with engineer

Optimization code (GA)

	H	G	F	E	D	C	B	A
8	4.10 U 23700	4.10 G 0	4.10 G 18900	4.10 G 20400	4.10 G 20300	4.10 G 0	4.10 U 11400	4.10 G 34100
9	4.10 G 0	4.10 U 15000	4.10 U 31700	4.10 U 29500	4.10 G 0	4.10 G 18800	4.10 U 14800	4.10 U 0
10	4.10 [illegible]	4.10 [illegible]	4.10 [illegible]	4.10 [illegible]	4.10 [illegible]	4.10 [illegible]	4.10 [illegible]	4.10 [illegible]
11	[illegible] G 20400	[illegible] U 29500	[illegible] G 0	[illegible] U 11300	[illegible] U 29600	[illegible] G 0	[illegible] U 13600	[illegible] U 31800
12	4.10 G 20300	4.10 G 0	4.10 G 18500	4.10 U 29500	4.10 G 0	4.10 G 18000	4.10 U 0	
13	4.10 G 0	4.10 G 18800	4.10 U 13900	4.10 G 0	4.10 G 17800	4.10 U 0	4.10 G 32900	
14	4.10 U 11300	4.10 U 14700	4.10 G 0	4.10 U 13500	4.10 U 0	4.10 G 32900		
15	4.10 G 34200	4.10 U 0	4.10 U 0	3.50 U 31900				

--Enrichment(wt%)
--Fuel Type(U/G=without Gd/(
--Burnup(MWd/t, BOC)

Radial Peaking factor		1.443	(≤1.450)
Cycle length	(MWd/t)	17429	
Maximum burnup	(MWd/t)	46989	(≤48000)
Mod. temp. coeff.	(pcm/C)	-0.5	(≤-0.5)
Radial power tilt	(%)	0.22	

Engineer

	H	G	F	E	D	C	B	A
8	4.10 U 23700	4.10 U 14700	4.10 G 0	4.10 U 29500	4.10 G 0	4.10 U 29700	4.10 U 15000	4.10 G 34100
9	4.10 U 14800	4.10 U 14800	4.10 U 29500	4.10 G 0	4.10 G 20400	4.10 G 18300	4.10 G 0	4.10 U 0
10	4.10 G 0	4.10 U 2950(	4.10 [illegible]	3.50 [illegible]	4.10 [illegible]	4.10 G 7800	4.10 U 13600	4.10 U 0
11	4.10 U 29500	4.10 G 0	[illegible] U 31800	[illegible] U 11400	[illegible] G 18800	.10 U 14000	4.10 G 0	4.10 U 31700
12	4.10 G 0	4.10 G 20300	4.10 G 0	4.10 G 18800	4.10 G 0	4.10 G 18600	4.10 U 0	
13	4.10 U 29500	4.10 G 18900	4.10 G 18000	4.10 U 13900	4.10 G 18500	4.10 U 0	4.10 G 32900	
14	4.10 U 15000	4.10 G 0	4.10 U 13500	4.10 G 0	4.10 U 0	4.10 G 32900		
15	4.10 G 34200	4.10 U 0	4.10 U 0	4.10 U 31700				

--Enrichment(wt%)
--Fuel Type(U/G=without Gd/(
--Burnup(MWd/t, BOC)

Radial Peaking factor		1.445	(≤1.450)
Cycle length	(MWd/t)	17163	
Maximum burnup	(MWd/t)	47104	(≤48000)
Mod. temp. coeff.	(pcm/C)	-0.5	(≤-0.5)
Radial power tilt	(%)	0.14	

Single-cycle vs multi-cycle optimization

- Loading pattern optimization is difficult even for a single cycle, due to huge design space and many constraints and objectives.
- However, since a fuel assembly stays in-core for a few to several cycles, we should consider multi-cycle to improve “overall” fuel cycle cost.
- As previously discussed, the single cycle optimization may have the adverse effect on the multi-cycle optimization.
- We should be very careful on the multi-cycle coupling effect, which cannot be captured by the single cycle optimization.

Summary

Summary of this lecture

- In-core fuel management tasks in PWR
- Basic theory of in-core fuel management
  - Linear reactivity model
  - Cycle length, discharge burnup and batch fraction
  - Analysis of power sharing
- Loading pattern optimization methods
  - Objectives and constraints
  - Simulated annealing
  - Genetic algorithms
  - Tabu-search
  - Heuristics
  - Example of applications

Supplements

Comparison with successive single-cycle and “true” multi-cycle optimum power sharing

- Power sharing at each cycle (max. cycle length)

Cycle	Successive single cycle			True multi cycle		
	new	1ce	2ce	new	1ce	2ce
1	1.436	1.000	0.564	1.000	1.000	1.000
2	1.661	0.800	0.539	1.000	1.000	1.000
3	1.713	0.864	0.424	0.996	1.002	1.001
4	1.714	0.856	0.431	1.110	0.946	0.944
5	1.714	0.857	0.428	1.479	0.948	0.574

Comparison with successive single-cycle and "true" multi-cycle optimum power sharing

- Cycle length at each cycle (max. cycle length)

Cycle	Successive single cycle	True multi cycle
1	17.183	14.998
2	14.341	15.006
3	14.023	14.971
4	14.004	15.757
5	14.001	16.498
Sum	73.550	77.229

Comparison with successive single-cycle and "true" multi-cycle optimum power sharing

- Discharge burnup at each cycle (max. cycle length)

Cycle	Successive single cycle	True multi cycle
1	39.684	45.001
2	39.910	45.000
3	42.097	44.990
4	41.959	44.887
5	42.004	39.295
Sum	205.653	219.174

Comparison with successive single-cycle and "true" multi-cycle optimum power sharing

- Power sharing of each batch (max. discharge burnup)

Cycle	Successive single cycle			True multi cycle		
	new	1ce	2ce	new	1ce	2ce
1	1.146	0.573	1.281	1.000	1.000	1.000
2	1.036	0.518	1.446	1.000	1.000	1.000
3	1.036	0.518	1.446	0.993	1.004	1.004
4	1.036	0.518	1.446	0.820	1.097	1.083
5	1.036	0.518	1.446	1.062	0.670	1.268

Comparison with successive single-cycle and "true" multi-cycle optimum power sharing

- Cycle length at each cycle (max. max. discharge burnup)

Cycle	Successive single cycle	True multi cycle
1	13.090	15.000
2	14.478	14.997
3	14.478	14.947
4	14.478	13.526
5	14.478	14.117
Sum	71.003	72.588

Comparison with successive single-cycle and "true" multi-cycle optimum power sharing

- Discharge burnup at each cycle (max. max. discharge burnup)

Cycle	Successive single cycle	True multi cycle
1	46.771	44.998
2	43.434	45.005
3	43.435	44.997
4	43.435	44.647
5	43.434	47.569
Sum	220.508	227.216